# General Physics (PHY 2130)

## Lecture 3

- Motion in one dimension
  - Position and displacement
  - Velocity
    - average
    - instantaneous
  - Acceleration
    - motion with constant acceleration

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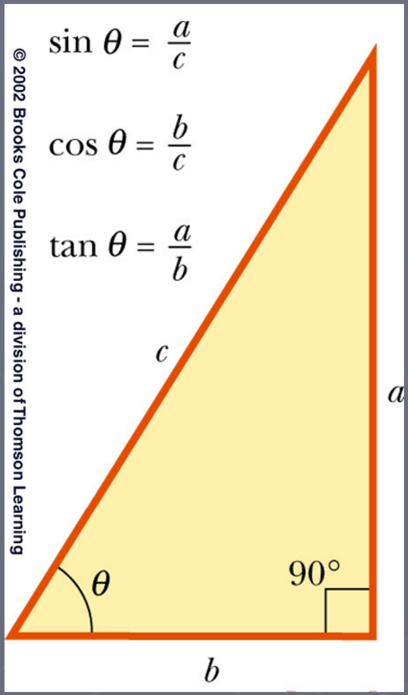
http://www.physics.wayne.edu/~apetrov/PHY2130/

# Lightning Review




Last lecture:

1. Math review: order of magnitude estimates, etc.
2. Physics introduction: units, measurements, etc.

Review Problem: How many beats would you detect if you take someone's pulse for 10 sec instead of a minute?

Hint: Normal heartbeat rate is 60 beats/minute.

Solution: recall that 1 minute = 60 seconds, so

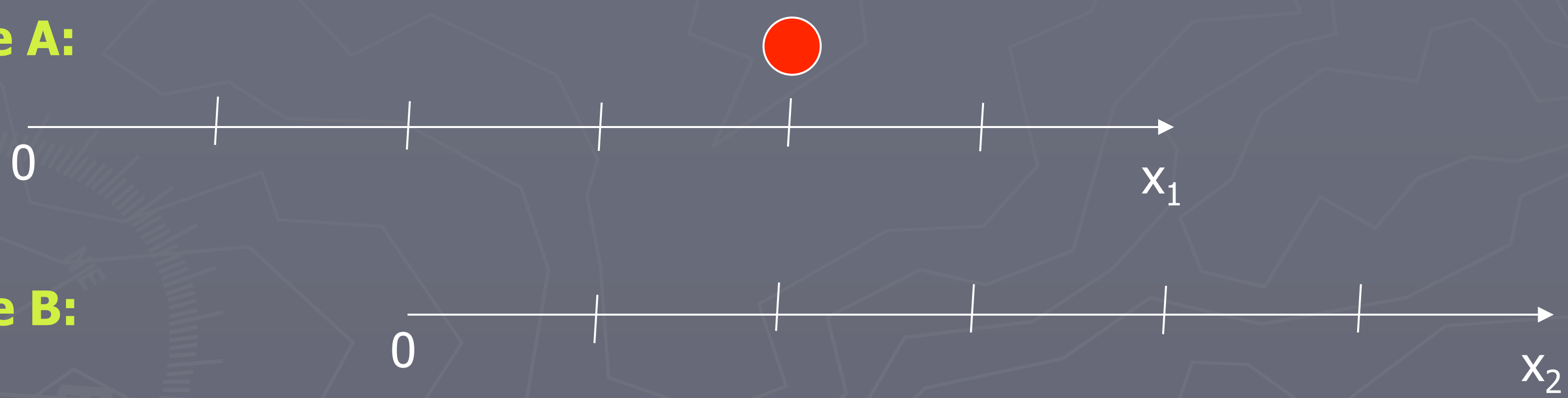
$$\frac{60\ beats}{\cancel{\mathrm{min}}} \times \frac{1\ \cancel{\mathrm{min}}}{60\ \cancel{\mathrm{sec}}} \times 10\ \cancel{\mathrm{sec}} = 10\ beats$$

# Position and Displacement

The **position** (x) of an object describes its location relative to some origin or other reference point (frame of reference).

**Frame A:**

**Frame B:**




The position of the red ball differs in the two shown coordinate systems.

# Example:

(a)


The position of the ball is $x = +2\ cm$

(b)

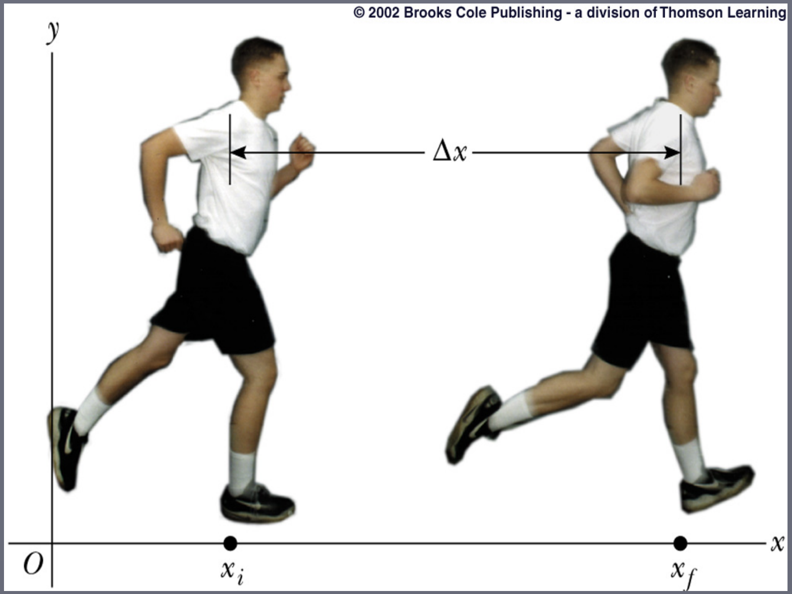

The position of the ball is $x = -2\ cm$

Note: (a) the "+" indicates the direction to the right of the origin;

(b) the "-" indicates the direction to the left of the origin.

# Position and Displacement

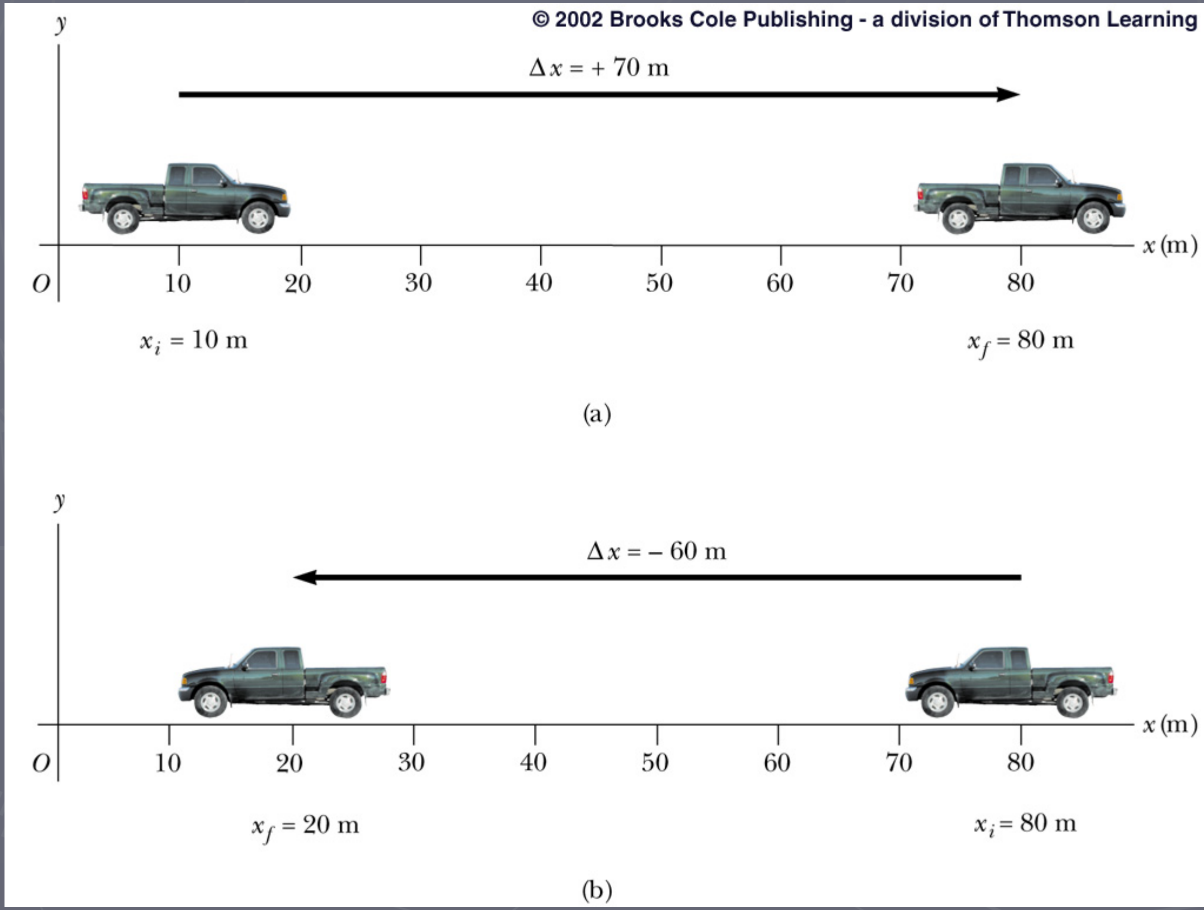
- Position is defined in terms of a **frame of reference**
  - One dimensional, so generally the x- or y-axis
- Displacement measures the change in position
  - Represented as $\Delta x$ (if horizontal) or $\Delta y$ (if vertical)
  - Needs directional information (i.e. "vector quantity")
    - + or - is generally sufficient to indicate direction for one-dimensional motion




| Units | |
|---|---|
| SI | Meters (m) |
| CGS | Centimeters (cm) |
| US Cust | Feet (ft) |

# Displacement

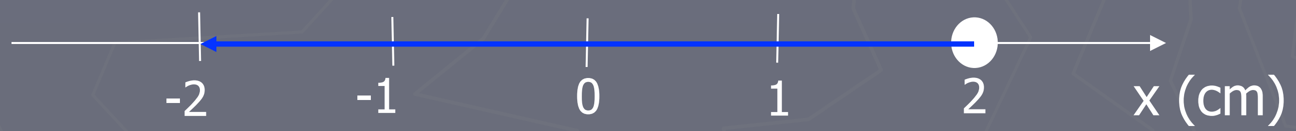
- Displacement measures the change in position
  - represented as $\Delta x$ or $\Delta y$




$$\Delta x_1 = x_f - x_i$$
$$= 80\,m - 10\,m$$
$$= \underline{+70\,m}$$ ✓

$$\Delta x_2 = x_f - x_i$$
$$= 20\,m - 80\,m$$
$$= \underline{-60\,m}$$ ✓

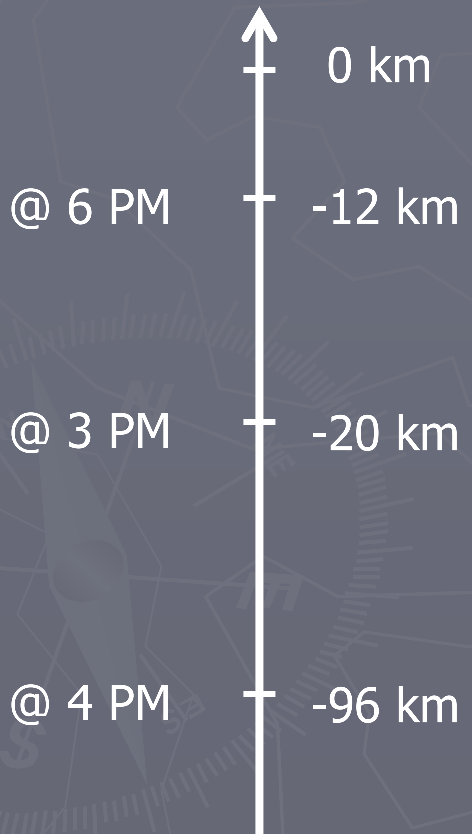
Example: A ball is initially at x = +2 cm and is moved to x = -2 cm.  What is the displacement of the ball?




$$\Delta x = x_f - x_i$$
$$= -2\text{ cm} - 2\text{ cm}$$
$$= -4\text{ cm}$$

**Example:** At 3 PM a car is located 20 km south of its starting point. One hour later its is 96 km farther south. After two more hours it is 12 km south of the original starting point.

(a) What is the displacement of the car between 3 PM and 6 PM?




**Solution:**

Use a coordinate system where north is positive.

Then, $x_i$ = –20 km and $x_f$ = –12 km

$$\Delta x = x_f - x_i$$
$$= -12\text{ km} - \left(-20\text{ km}\right) = +8\text{ km}$$

Notice the signs of $x_i$ and $x_f$ !!!

## Example continued

(b) What is the displacement of the car from the starting point to the location at 4 pm?

$x_i$ = 0 km and $x_f$ = –96 km

$$\begin{aligned}\Delta x &= x_f - x_i \\ &= -96\text{ km} - \left(0\text{ km}\right) = -96\text{ km}\end{aligned}$$

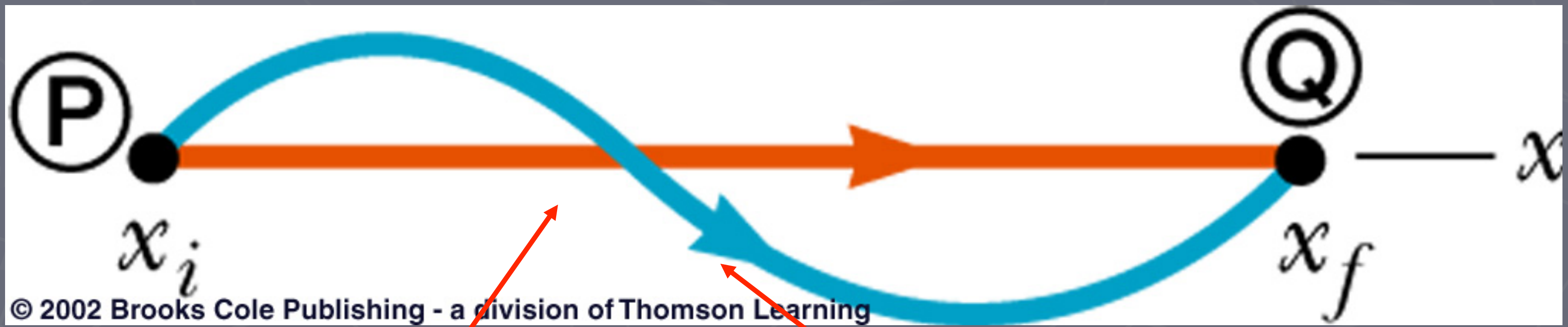
(c) What is the displacement of the car from 4 PM to 6 PM?

$x_i$ = –96 km and $x_f$ = –12 km

$$\begin{aligned}\Delta x &= x_f - x_i \\ &= -12\text{ km} - \left(-96\text{ km}\right) = +84\text{ km}\end{aligned}$$

# Distance or Displacement?

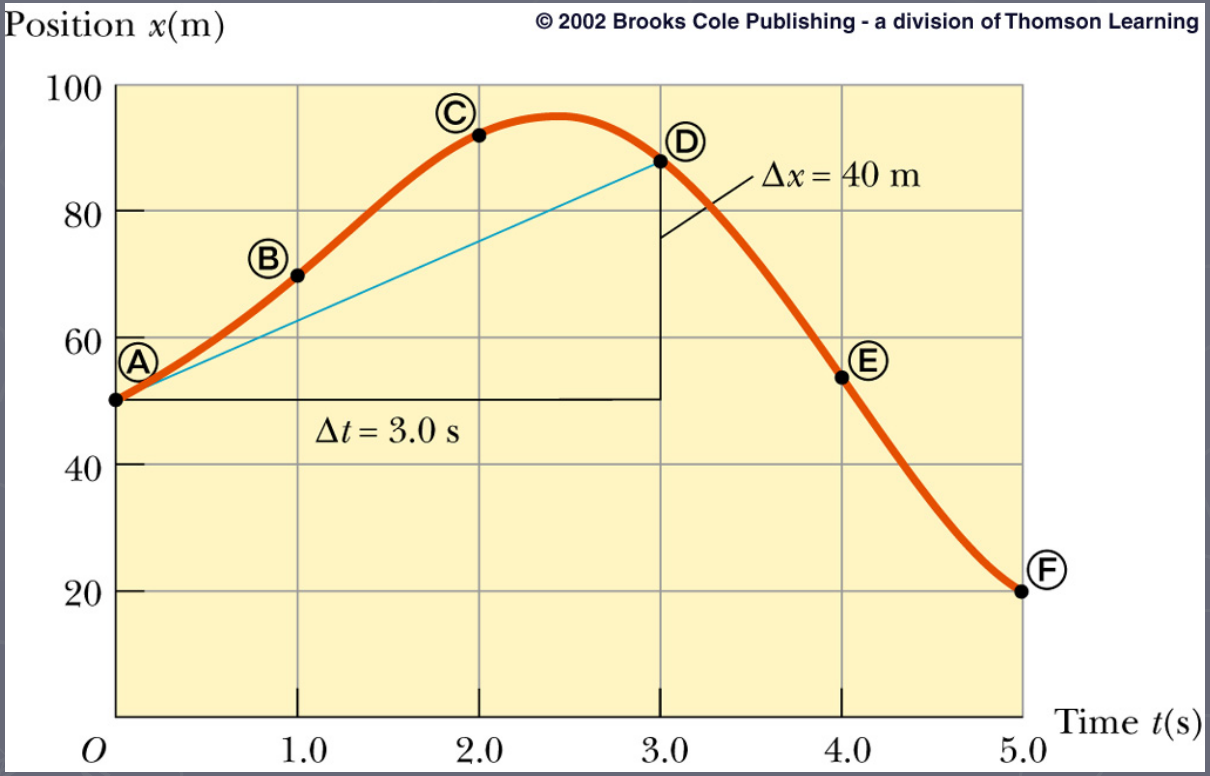
- Distance may be, but is not necessarily, the magnitude of the displacement




Displacement
(yellow line)

Distance
(blue line)

# Position-time graphs




- Note: position-time graph is not necessarily a straight line, even though the motion is along x-direction

# Average Velocity

- It takes time for an object to undergo a displacement
- The **average velocity** is rate at which the displacement occurs

$$\boxed{\vec{v}_{average} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\vec{x}_f - \vec{x}_i}{\Delta t}}$$

- Direction will be the same as the direction of the displacement ($\Delta t$ is always positive)

# More About Average Velocity

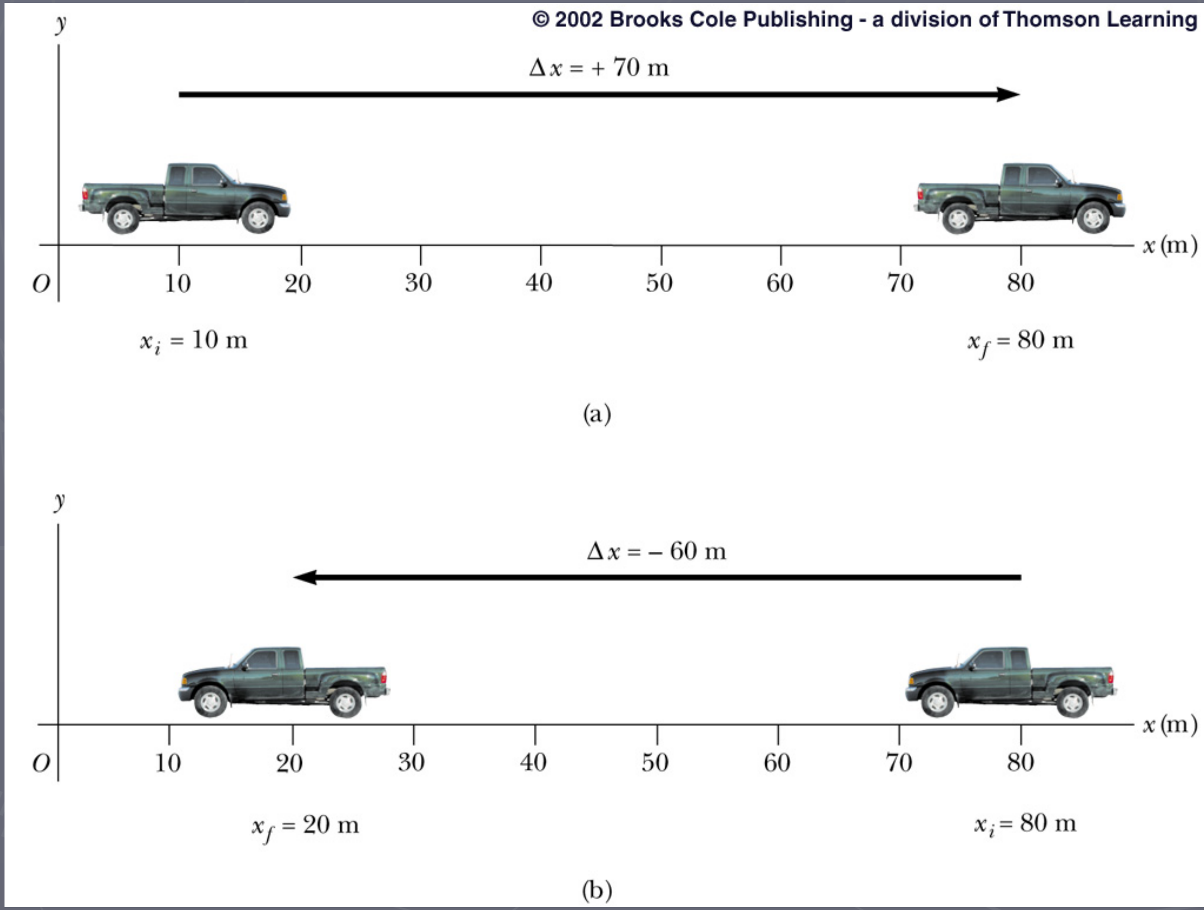
- Units of velocity:

| Units | |
|---|---|
| SI | Meters per second (m/s) |
| CGS | Centimeters per second (cm/s) |
| US Customary | Feet per second (ft/s) |

- Note: other units may be given in a problem, but generally will need to be converted to these

# Example:

Suppose that in both cases truck covers the distance in 10 seconds:




$$\vec{v}_{1\,average} = \frac{\Delta \vec{x}_1}{\Delta t} = \frac{+70m}{10s}$$
$$= \underline{+7\, m/s}$$

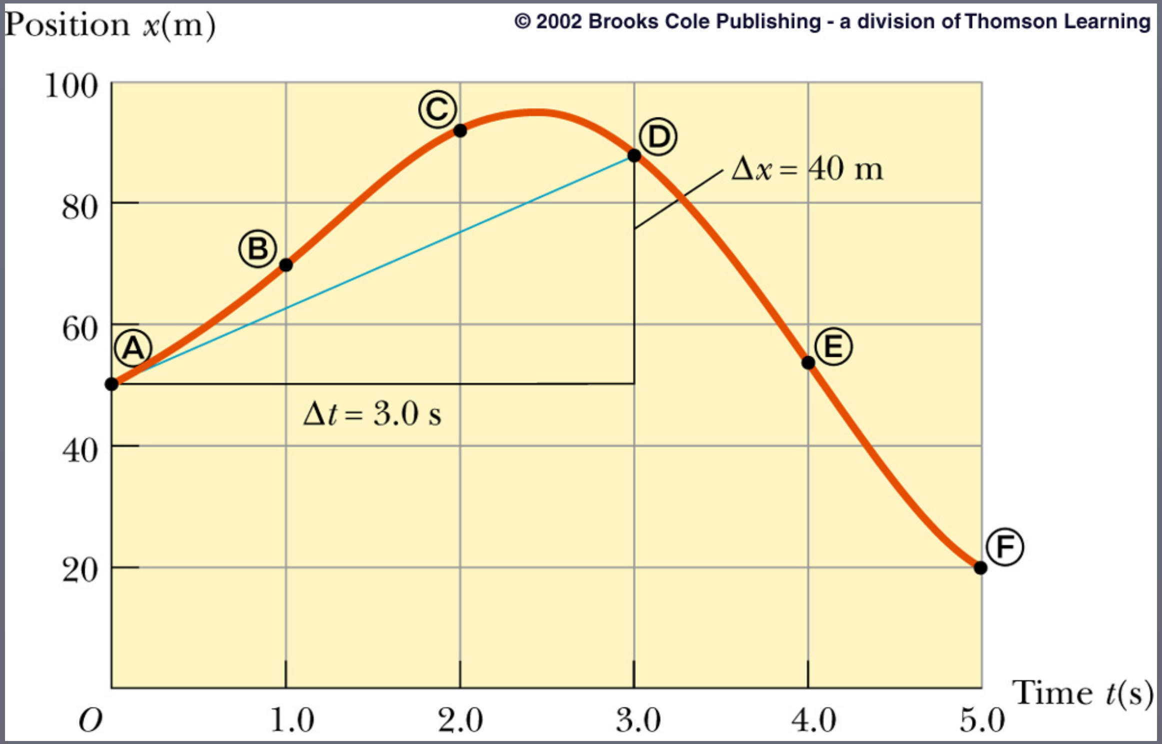
$$\vec{v}_{2\,average} = \frac{\Delta \vec{x}_2}{\Delta t} = \frac{-60m}{10s}$$
$$= \underline{-6\, m/s}$$

# Speed

- Speed is a scalar quantity (no information about sign/direction is need)
  - same units as velocity
  - Average speed = total distance / total time
- Speed is the magnitude of the velocity

# Graphical Interpretation of Average Velocity

- Velocity can be determined from a position-time graph




$$\vec{v}_{average} = \frac{\Delta \vec{x}}{\Delta t} = \frac{+40m}{3.0s}$$

$$= \underline{+13\, m/s}$$

- Average velocity equals the slope of the line joining the initial and final positions

# Instantaneous Velocity

- Instantaneous velocity is defined as the limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero

$$\boxed{\vec{v}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{x}_f - \vec{x}_i}{\Delta t}}$$

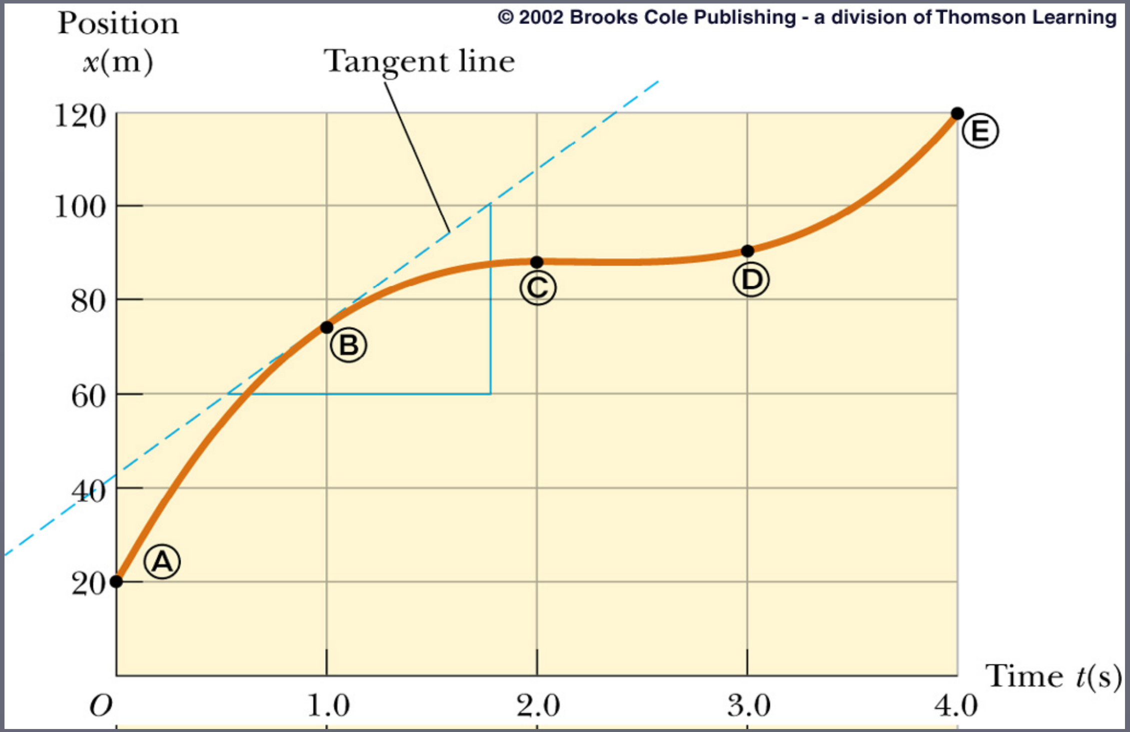
- The instantaneous velocity indicates what is happening at every point of time

# Uniform Velocity

- Uniform velocity is constant velocity
- The instantaneous velocities are always the same
  - All the instantaneous velocities will also equal the average velocity

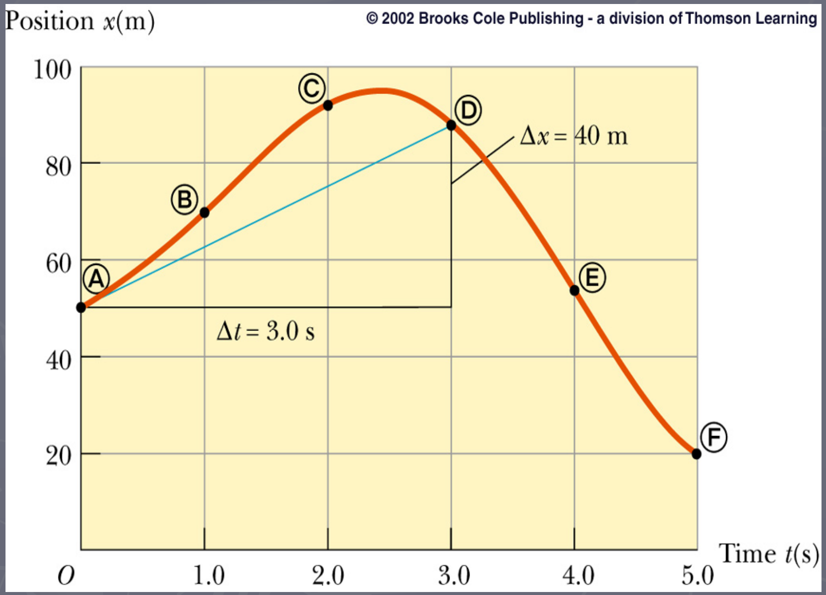
# Graphical Interpretation of Instantaneous Velocity

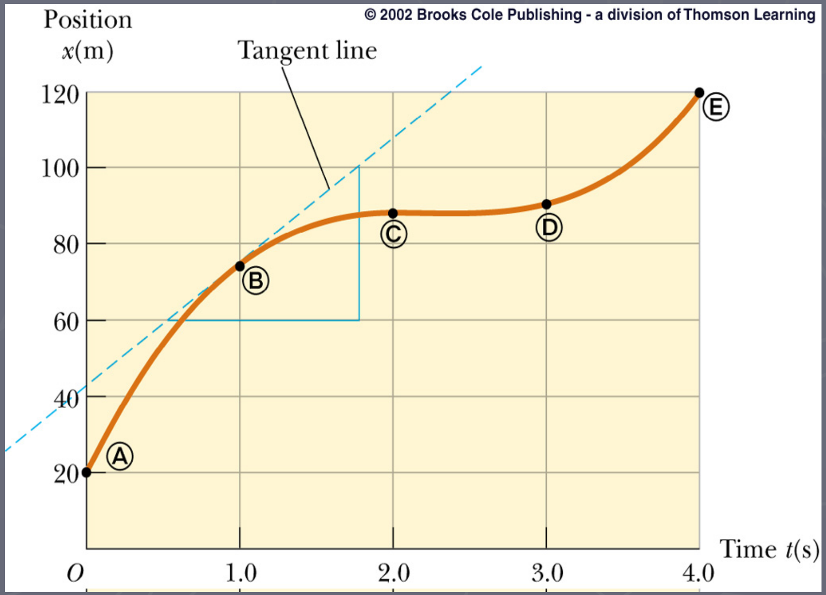
- Instantaneous velocity is the slope of the tangent to the curve at the time of interest




- The instantaneous speed is the magnitude of the instantaneous velocity

# Average vs Instantaneous Velocity




Average velocity

Instantaneous velocity

# Average Acceleration

- Changing velocity (non-uniform) means an acceleration is present
- Average acceleration is the rate of change of the velocity

$$\boxed{\vec{a}_{average} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}}$$

- Average acceleration is a vector quantity (i.e. described by both magnitude and direction)

# Average Acceleration

- When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
- When the sign of the velocity and the acceleration are opposite, the speed is decreasing

| Units | |
|---|---|
| SI | Meters per second squared ($m/s^2$) |
| CGS | Centimeters per second squared ($cm/s^2$) |
| US Customary | Feet per second squared ($ft/s^2$) |

# Instantaneous and Uniform Acceleration

- Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero

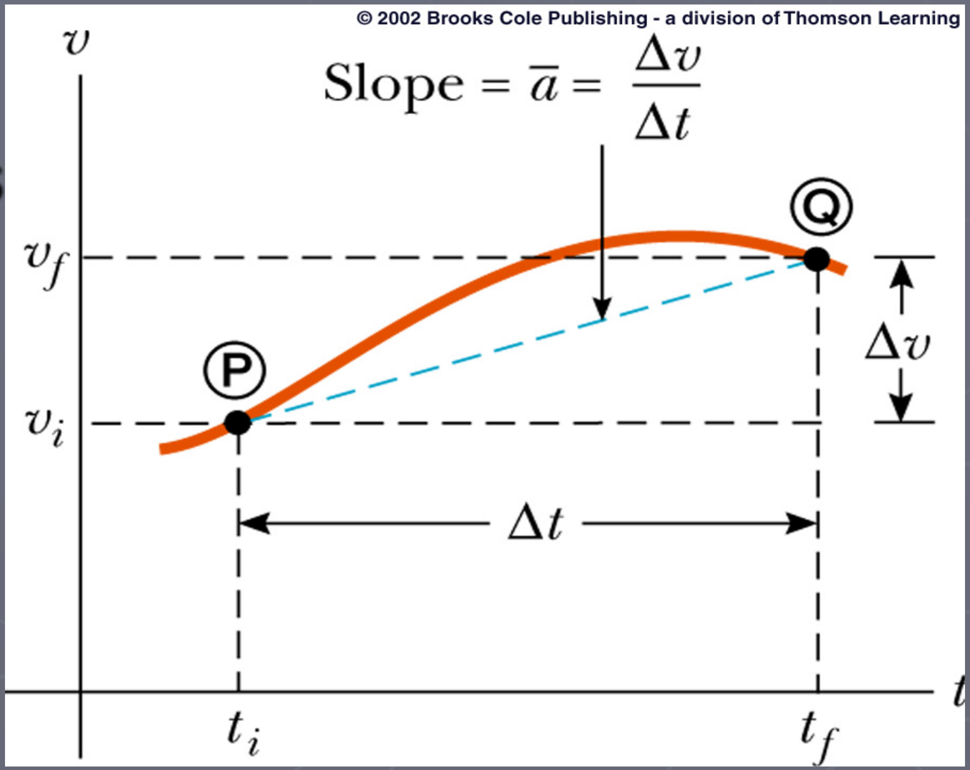
$$\boxed{\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}_f - \vec{v}_i}{\Delta t}}$$

- When the instantaneous accelerations are always the same, the acceleration will be uniform
  - The instantaneous accelerations will all be equal to the average acceleration

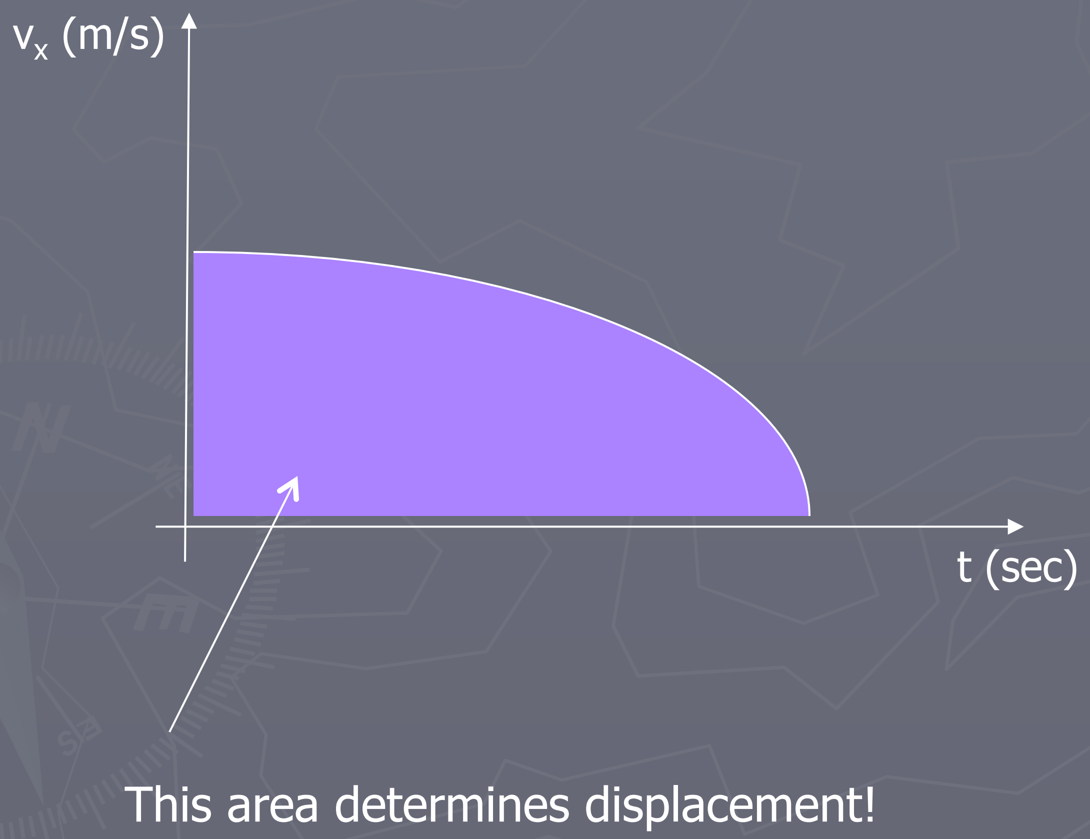
# Graphical Interpretation of Acceleration

- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph

- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

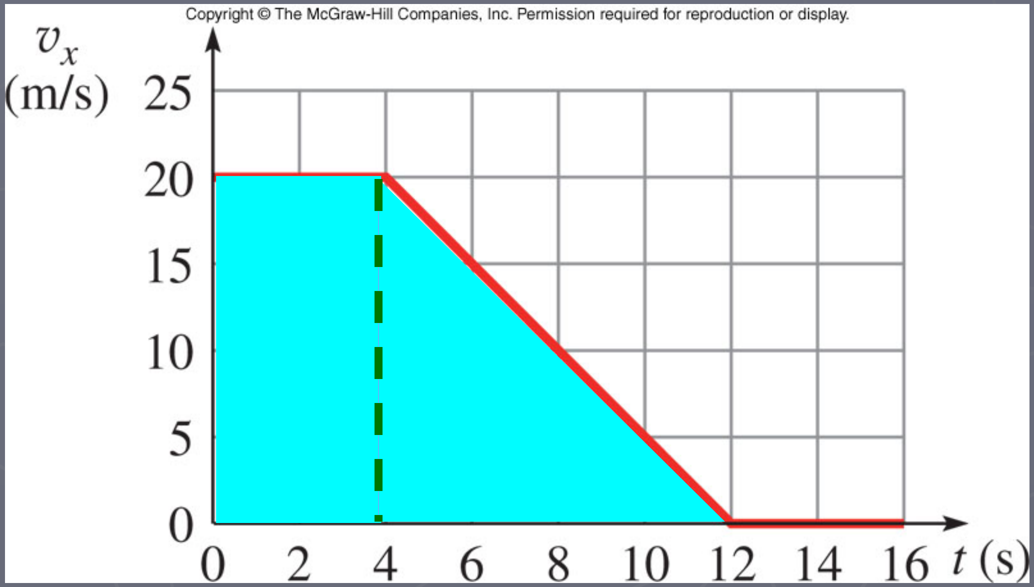


NEAT OBSERVATION: the area under a velocity versus time graph (between the curve and the time axis) gives the displacement in a given interval of time.



Example: Speedometer readings are obtained and graphed as a car comes to a stop along a straight-line path. How far does the car move between t = 0 and t = 16 seconds?

Solution: since there is not a reversal of direction, the area between the curve and the time axis will represent the distance traveled.



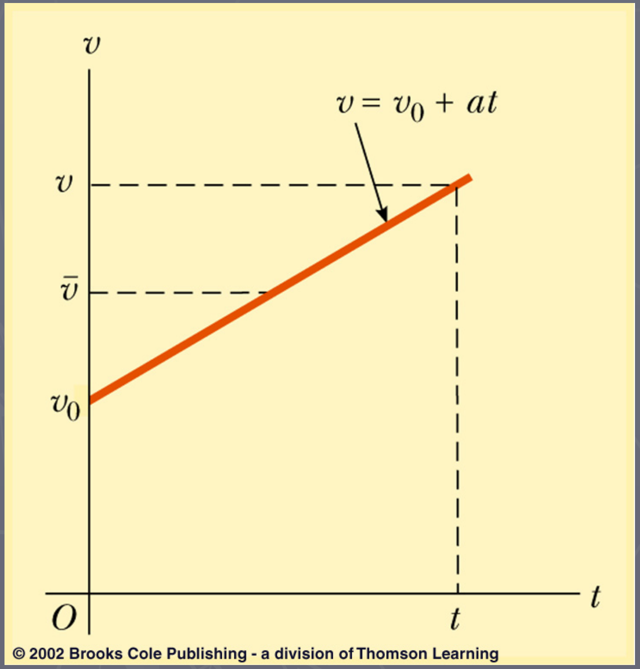

The rectangular portion has an area of Lw = (20 m/s)(4 s) = 80 m.

The triangular portion has an area of ½bh = ½(8 s) (20 m/s) = 80 m.

Thus, the total area is 160 m. This is the distance traveled by the car.

# One-dimensional Motion With Constant Acceleration

- If acceleration is uniform (i.e. $\bar{a} = a$):




$$a = \frac{v_f - v_o}{t_f - t_0} = \frac{v_f - v_o}{t}$$

thus:

$$\boxed{v_f = v_o + at}$$

- Shows velocity as a function of acceleration and time

# One-dimensional Motion With Constant Acceleration

- Used in situations with uniform acceleration

$$v_f = v_o + at$$

$$\Delta x = v_{average} t = \left( \frac{v_o + v_f}{2} \right) t$$

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

$$v_f^2 = v_o^2 + 2a\Delta x$$

Velocity changes uniformly!!!

# Notes on the equations

$$\Delta x = v_{average}\, t = \left(\frac{v_o + v_f}{2}\right) t$$

- Gives displacement as a function of velocity and time

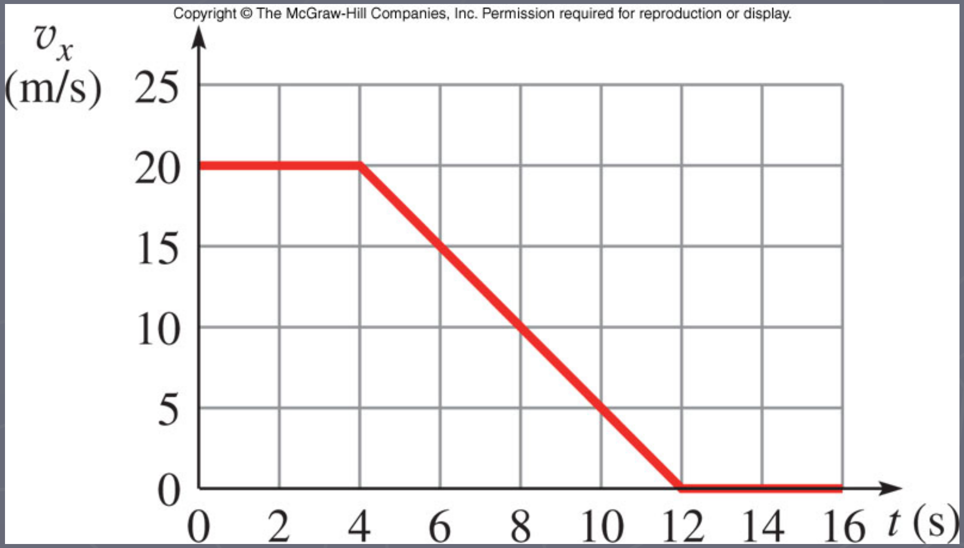
$$\Delta x = v_o t + \frac{1}{2} a t^2$$

- Gives displacement as a function of time, velocity and acceleration

$$v_f^2 = v_o^2 + 2a\Delta x$$

- Gives velocity as a function of acceleration and displacement

Example: The graph shows speedometer readings as a car comes to a stop. What is the magnitude of the acceleration at t = 7.0 s?




The slope of the graph at t = 7.0 sec is

$$\left|a_{\text{av}}\right| = \left|\frac{\Delta v_x}{\Delta t}\right| = \left|\frac{v_2 - v_1}{t_2 - t_1}\right| = \left|\frac{(0-20)\,\text{m/s}}{(12-4)\,\text{s}}\right| = 2.5\ \text{m/s}^2$$