General Physics (PHY 2130)

Lecture 4

- Motion in one dimension
  - Position and displacement
  - Velocity
    - average
    - instantaneous
  - Acceleration
    - motion with constant acceleration

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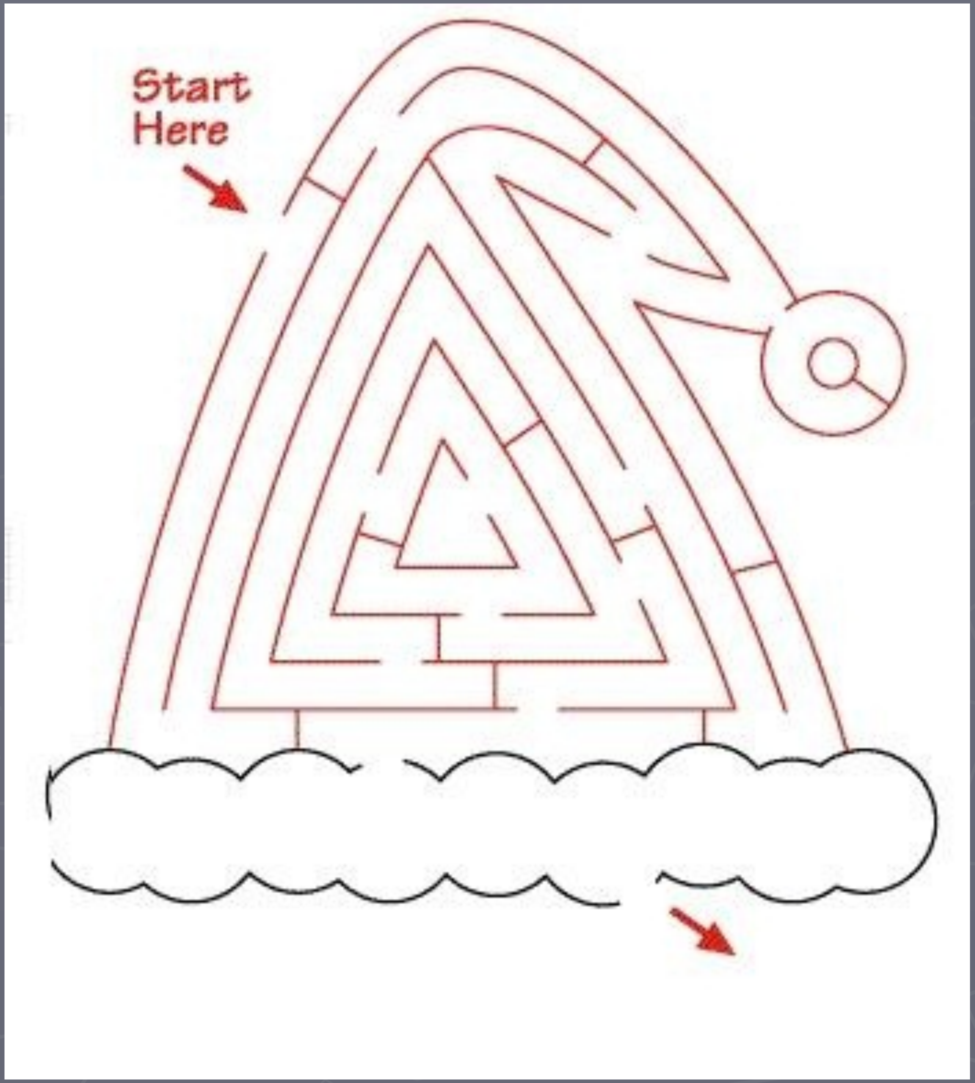
Lightning Review

Last lecture:

1. Motion in one dimension:
   - ✓ displacement: depends only on $x_f - x_i$
   - ✓ average velocity: displacement over time interval
   - ✓ instantaneous velocity: same as above for a very small time interval
   - ✓ average acceleration: velocity change over time interval
   - ✓ instantaneous acceleration: same as above for a very small time interval

Review: Displacement vs path



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Average Acceleration

- Changing velocity (non-uniform) means an acceleration is present
- Average acceleration is the rate of change of the velocity

$$\vec{a}_{average} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

- Average acceleration is a vector quantity (i.e. described by both magnitude and direction)

Average Acceleration

- When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
- When the sign of the velocity and the acceleration are opposite, the speed is decreasing

Units	
SI	Meters per second squared (m/s^2)
CGS	Centimeters per second squared (cm/s^2)
US Customary	Feet per second squared (ft/s^2)

Instantaneous and Uniform Acceleration

- Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero

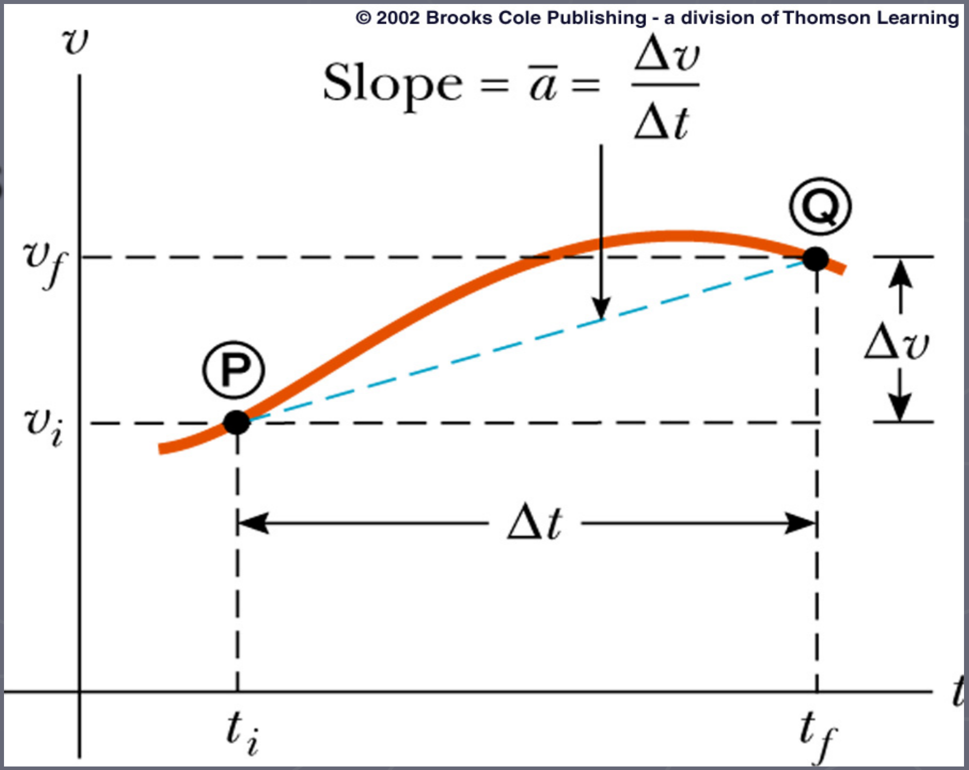
$$\vec{a}_{inst} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

- When the instantaneous accelerations are always the same, the acceleration will be uniform
  - The instantaneous accelerations will all be equal to the average acceleration

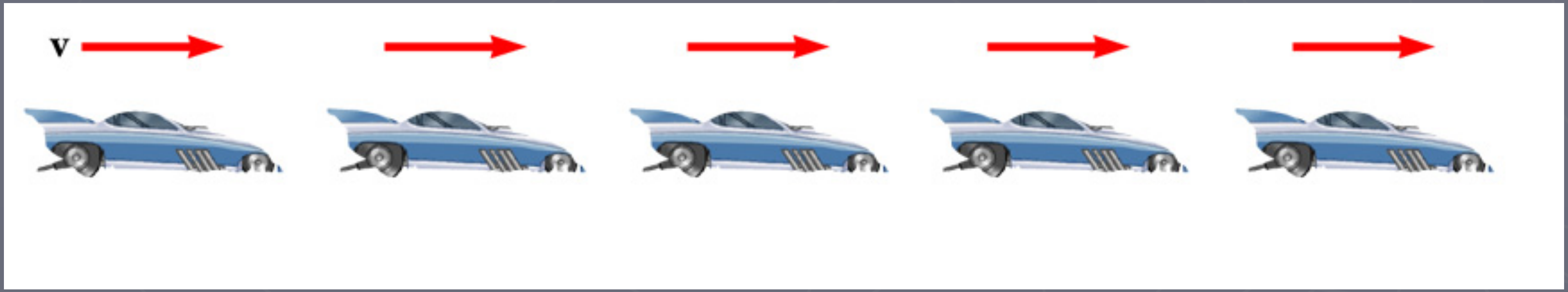
Graphical Interpretation of Acceleration

- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph

- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph

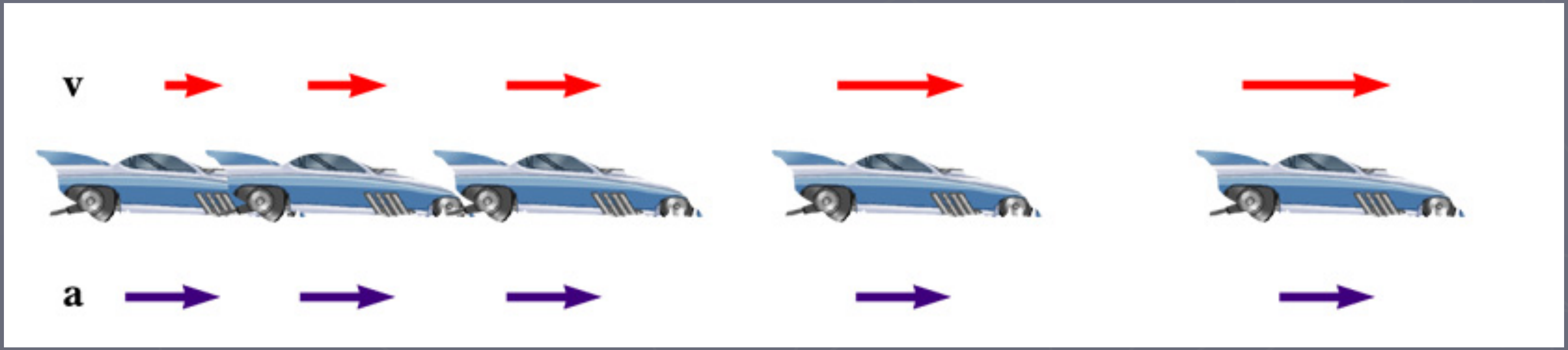


Example 1: Motion Diagrams



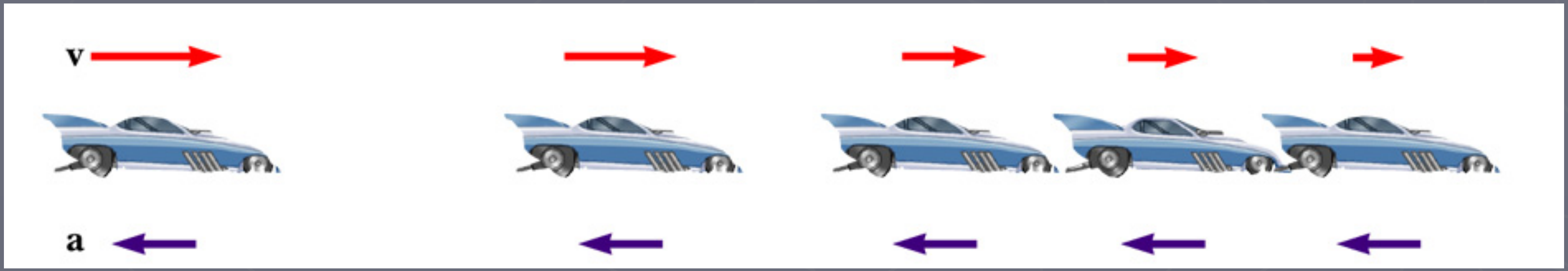
- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

Example 2:



- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)

Example 3:



- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)

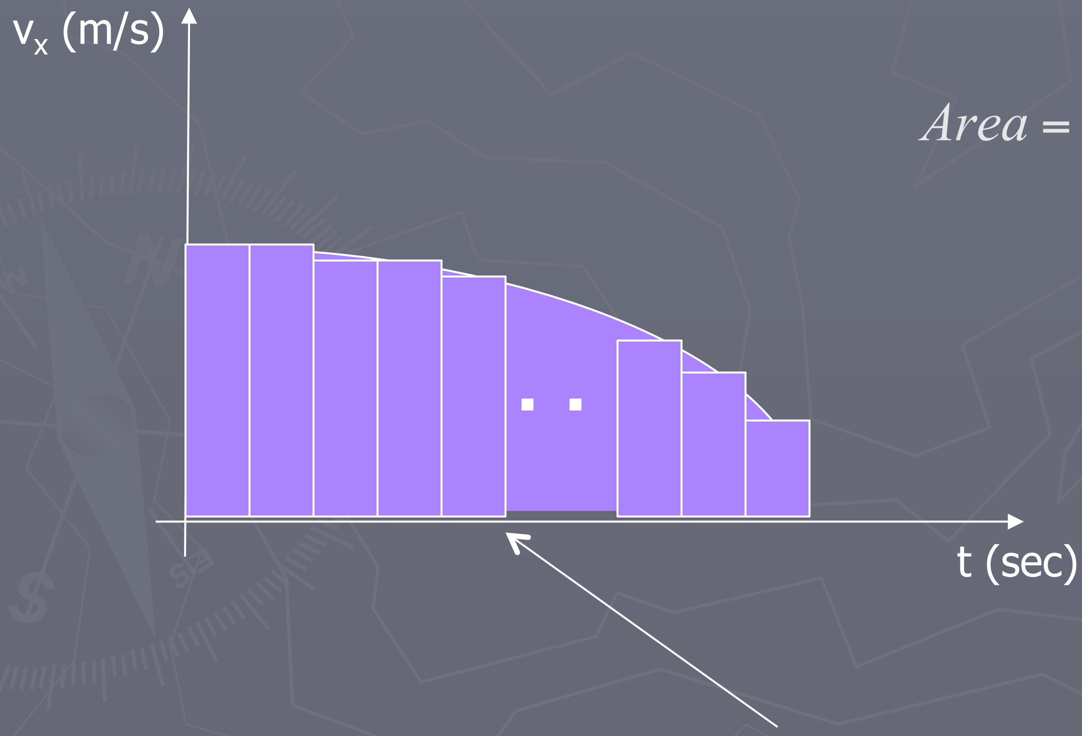
NEAT OBSERVATION: the area under a velocity versus time graph (between the curve and the time axis) gives the displacement in a given interval of time.

Why is that? For the constant velocity:

$$v_x = \frac{\Delta x}{\Delta t} \quad \Rightarrow \quad \Delta x = v_x \Delta t$$

Divide the graph into small rectangular shapes

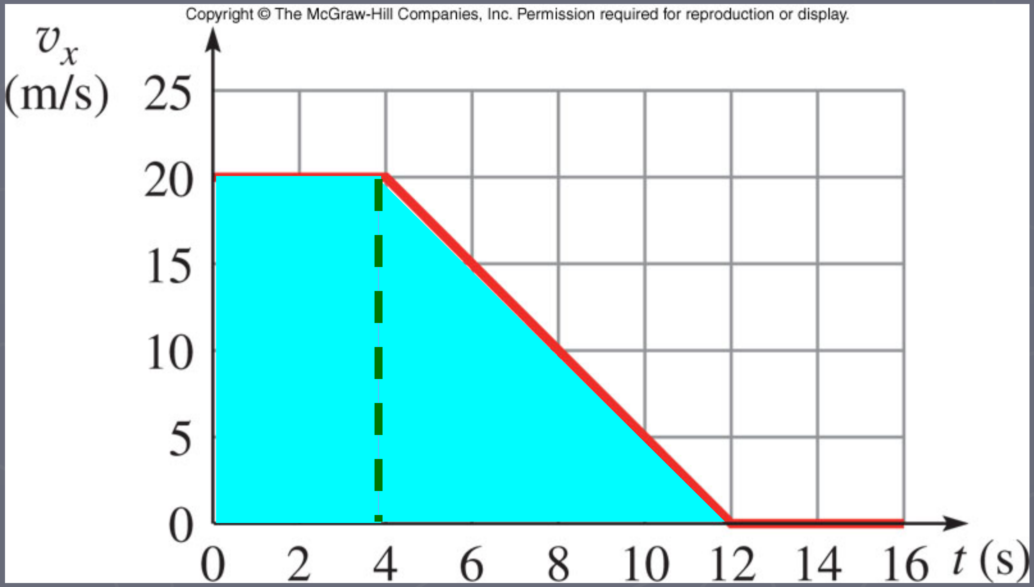
$$Area = \sum_i S_i = \sum_i (v_x)_i \Delta t_i = \sum_i \Delta x_i = \Delta x_{total}$$



This whole area determines displacement!

Example: Speedometer readings are obtained and graphed as a car comes to a stop along a straight-line path. How far does the car move between t = 0 and t = 16 seconds?

Solution: since there is not a reversal of direction, the area between the curve and the time axis will represent the distance traveled.



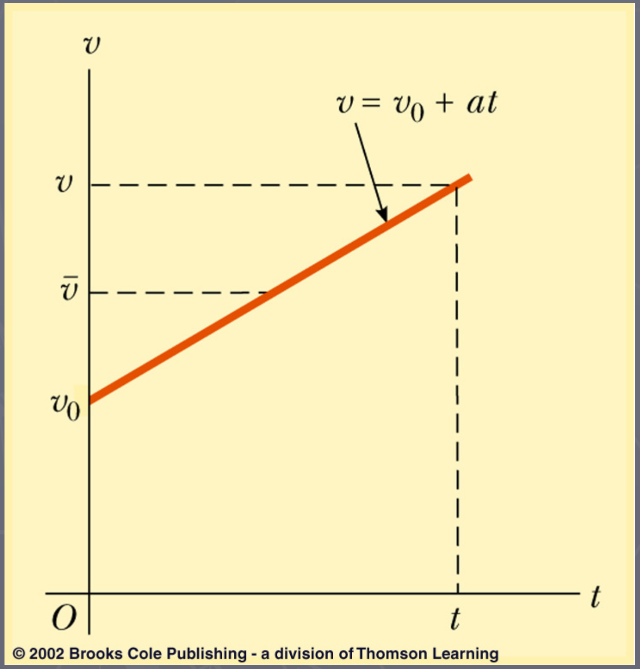
The rectangular portion has an area of Lw = (20 m/s)(4 s) = 80 m.

The triangular portion has an area of ½bh = ½(8 s) (20 m/s) = 80 m.

Thus, the total area is 160 m. This is the distance traveled by the car.

One-dimensional Motion With Constant Acceleration

- If acceleration is uniform (i.e. $\bar{a} = a$):



$$a = \frac{v_f - v_o}{t_f - t_0} = \frac{v_f - v_o}{t}$$

thus:

$$v_f = v_o + at$$

- Shows velocity as a function of acceleration and time

One-dimensional Motion With Constant Acceleration

- Used in situations with uniform acceleration

$$v_f = v_o + at$$

$$\Delta x = v_{average} t = \left(\frac{v_o + v_f}{2} \right) t$$

Velocity changes uniformly!!!

$$\Delta x = v_o t + \frac{1}{2} a t^2$$

$$v_f^2 = v_o^2 + 2a\Delta x$$

Notes on the equations

$$\Delta x = v_{average}\, t = \left(\frac{v_o + v_f}{2} \right) t$$

- Gives displacement as a function of velocity and time

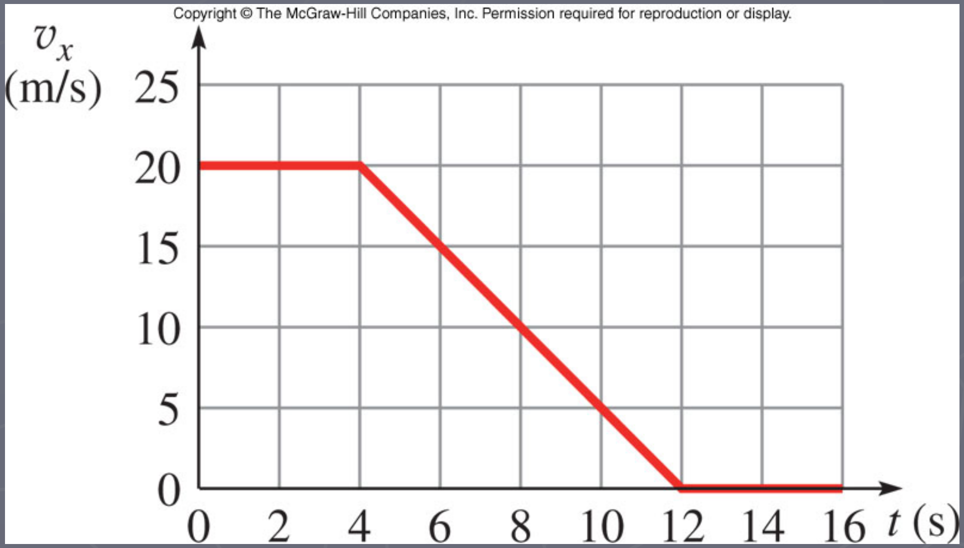
$$\Delta x = v_o t + \frac{1}{2} a t^2$$

- Gives displacement as a function of time, velocity and acceleration

$$v_f^2 = v_o^2 + 2a\Delta x$$

- Gives velocity as a function of acceleration and displacement

Example: The graph shows speedometer readings as a car comes to a stop. What is the magnitude of the acceleration at t = 7.0 s?



The slope of the graph at t = 7.0 sec is

$$|a_{av}| = \left|\frac{\Delta v_x}{\Delta t}\right| = \left|\frac{v_2 - v_1}{t_2 - t_1}\right| = \left|\frac{(0-20)\,\text{m/s}}{(12-4)\,\text{s}}\right| = 2.5\ \text{m/s}^2$$

Summary of kinematic equations

TABLE 2.3 **Equations for Motion in a Straight Line Under Constant Acceleration**

Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$\Delta x = \frac{1}{2}(v_0 + v)t$	Displacement as a function of velocity and time
$\Delta x = v_0 t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0^2 + 2a\,\Delta x$	Velocity as a function of displacement

Note: Motion is along the x axis. At $t = 0$, the velocity of the particle is v_0.

Free Fall

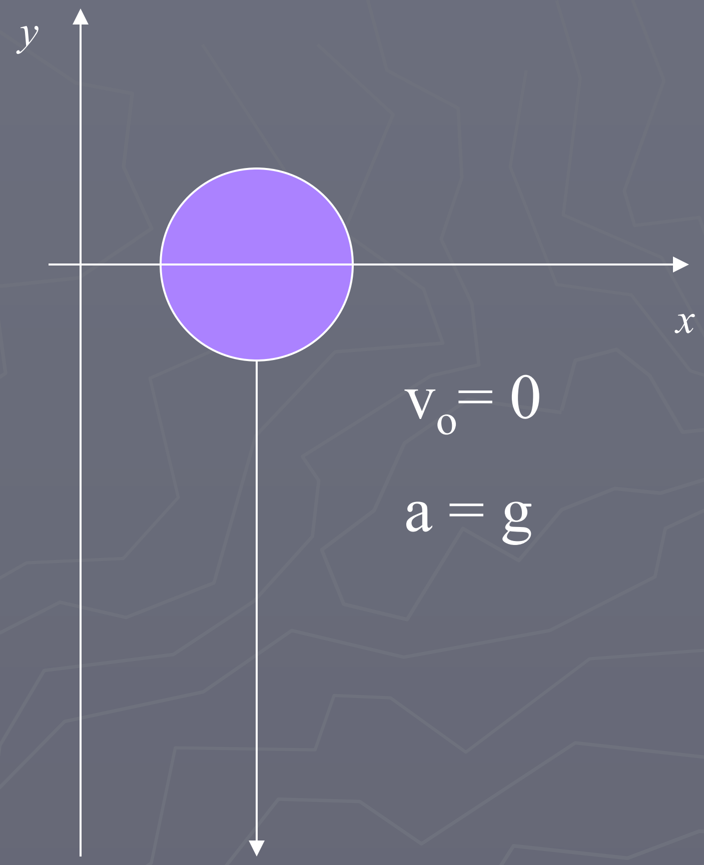
- All objects moving under the influence of only gravity are said to be in free fall
- All objects falling near the earth's surface fall with a constant acceleration
- This acceleration is called the acceleration due to gravity, and indicated by g

Acceleration due to Gravity

- Symbolized by g
- $g = 9.8\ m/s^2$ (can use $g = 10\ m/s^2$ for estimates)
- g is always directed downward
  - toward the center of the earth

Free Fall -- an Object Dropped

- Initial velocity is zero
- Frame: let up be positive
- Use the kinematic equations
  - Generally use y instead of x since vertical

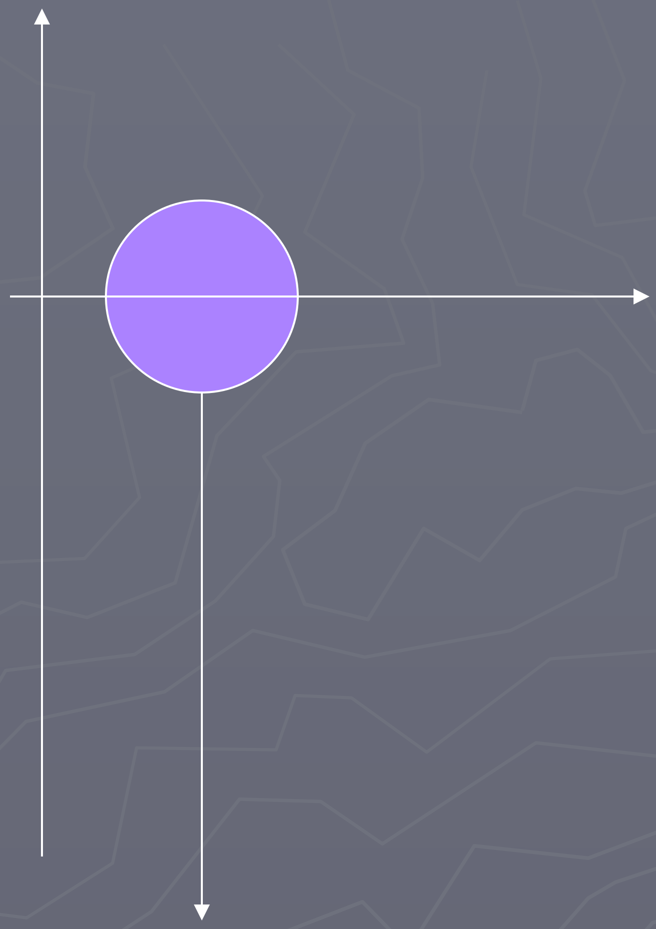


$$\Delta y = \frac{1}{2}at^2$$

$$a = -9.8\,m/s^2$$

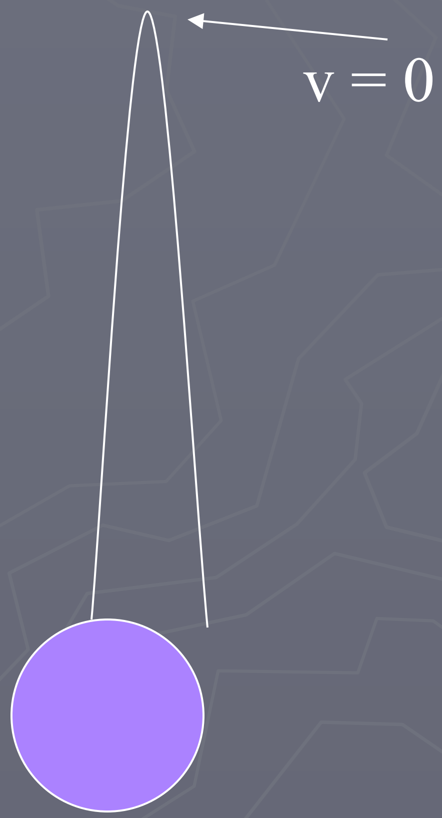
Free Fall -- an Object Thrown Downward

- $a = g$
  - With upward being positive, acceleration will be negative, $g = -9.8\ m/s^2$
- Initial velocity $\neq 0$
  - With upward being positive, initial velocity will be negative



Free Fall -- object thrown upward

- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- a = g everywhere in the motion
  - g is always downward, negative



Example: A penny is dropped from the observation deck of the Empire State Building 369 m above the ground. With what velocity does it strike the ground? Ignore air resistance.

Given:

$v_{iy} = 0$ m/s,

$a_y = -9.8$ m/s^2,

$\Delta y = -369$ m

Find:

$V_{yf} = ?$

Solution:

Use:

$$v_{fy}^2 = v_{iy}^2 + 2a_y \Delta y$$

$$= 2a_y \Delta y$$

$$v_{yf} = \sqrt{2a_y \Delta y}$$

$$v_{yf} = \sqrt{2a_y \Delta y} = \sqrt{2\left(-9.8\ \mathrm{m/s^2}\right)\left(-369\right)\mathrm{m}} = 85.0\ \mathrm{m/s}$$

Example continued:

How long does it take for the penny to strike the ground?

Given: v_{iy} = 0 m/s, a_y = -9.8 m/s^2, Δy = -369 m Unknown: Δt

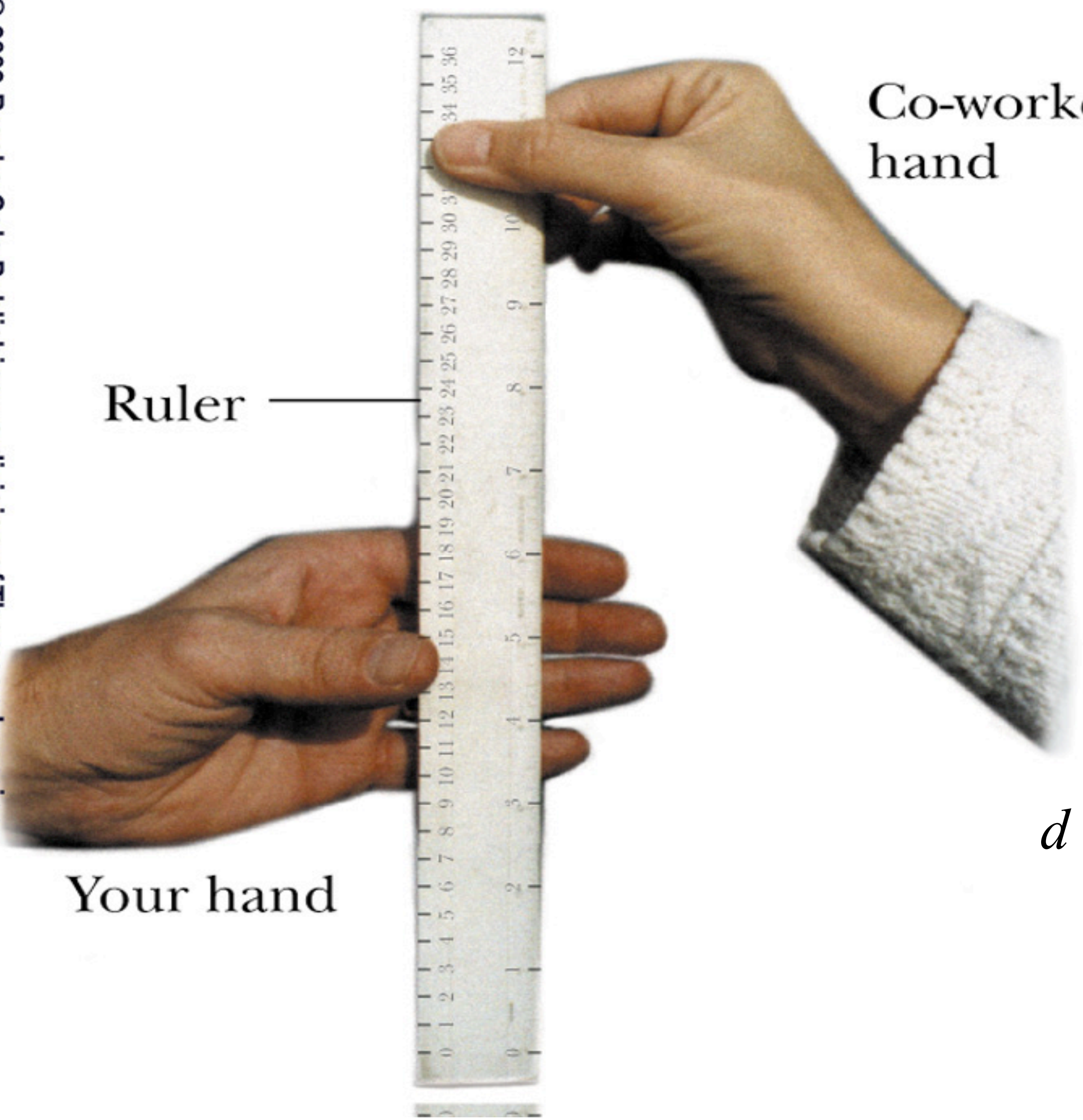
$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a_y \Delta t^2 = \frac{1}{2}a_y \Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = 8.7\,\mathrm{sec}$$

Thrown upward

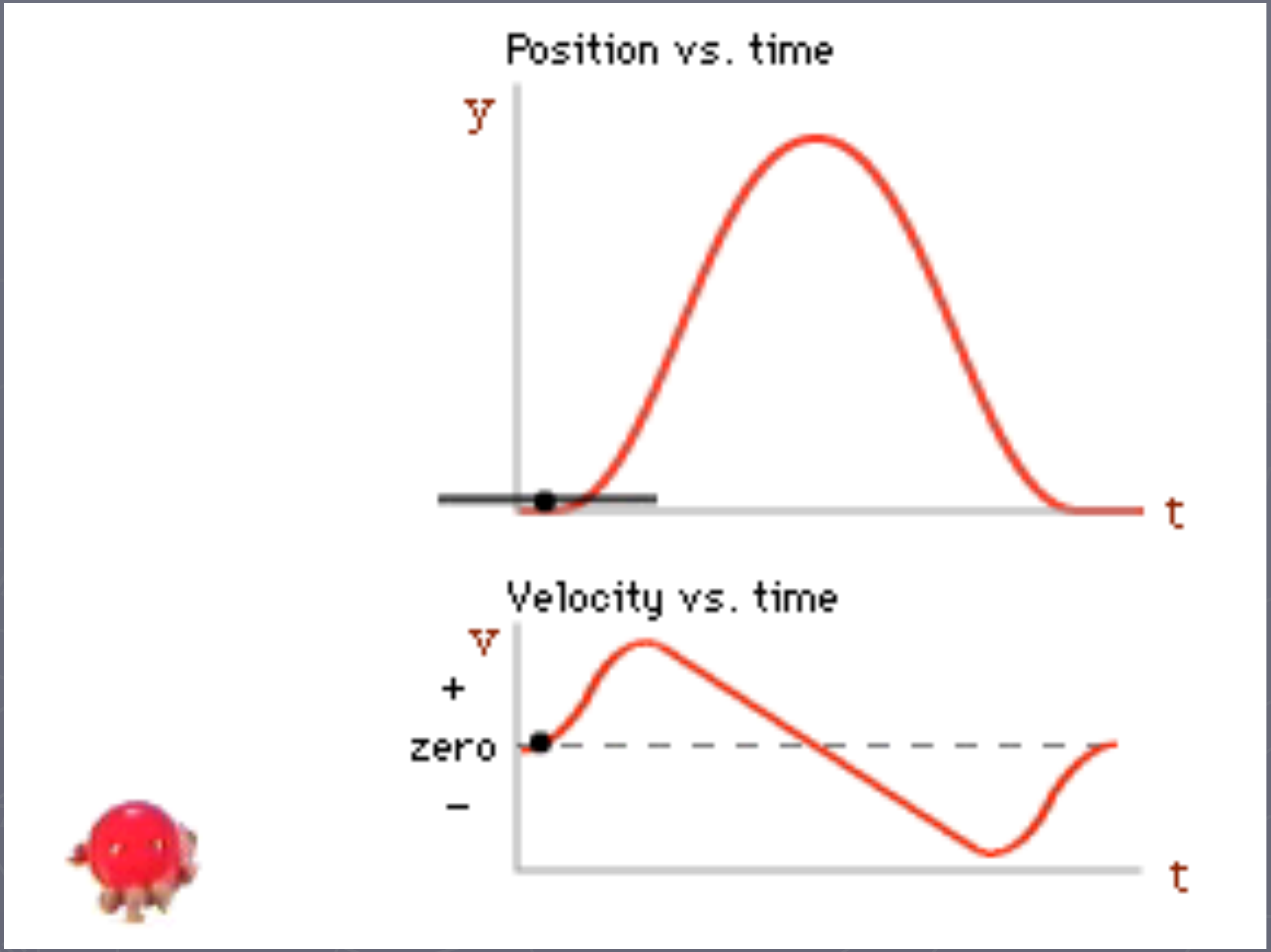
- The motion may be symmetrical
  - then $t_{up} = t_{down}$
  - then $v_f = -v_o$
- The motion may not be symmetrical
  - Break the motion into various parts
    - generally up and down

Fun QuickLab: Reaction time



$$d = \frac{1}{2} g t^2, \; g = 9.8\, m/s^2$$

$$t = \sqrt{\frac{2d}{g}}$$



Position vs. time
y
t
Velocity vs. time
v
+
zero
−
t