General Physics (PHY 2130)

Lecture 5

- Math review: vectors

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Lightning Review

Last lecture:

1. Motion in one dimension:
   - ✓ average acceleration: velocity change over time interval
   - ✓ instantaneous acceleration: same as above for a very small time interval
   - ✓ free fall: motion with constant acceleration due to gravity

Review Problem: You are throwing a ball straight up in the air. At the highest point, the ball's

(1) velocity and acceleration are zero
(2) velocity is nonzero but its acceleration is zero
(3) acceleration is nonzero, but its velocity is zero
(4) velocity and acceleration are both nonzero

Math Review: Coordinate Systems

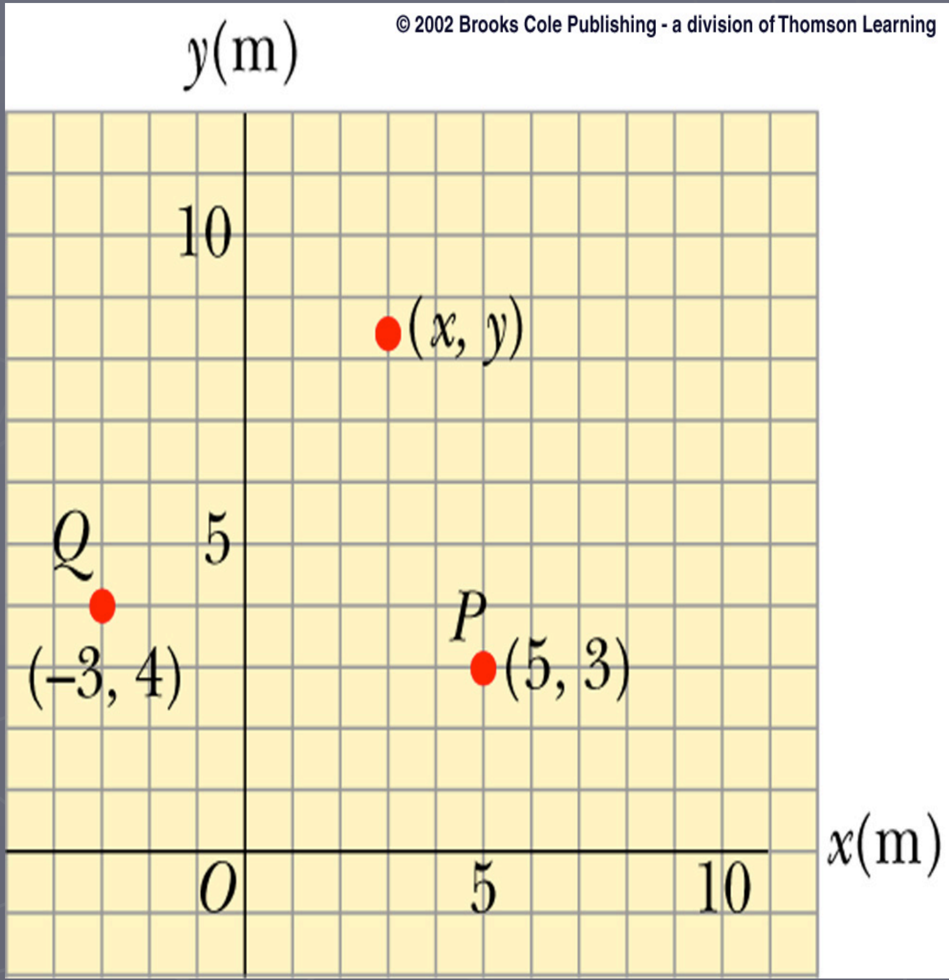
- Used to describe the position of a point in space
- Coordinate system (frame) consists of
  - a fixed reference point called the origin
  - specific axes with scales and labels
  - instructions on how to label a point relative to the origin and the axes

Types of Coordinate Systems

- Cartesian
- Plane polar

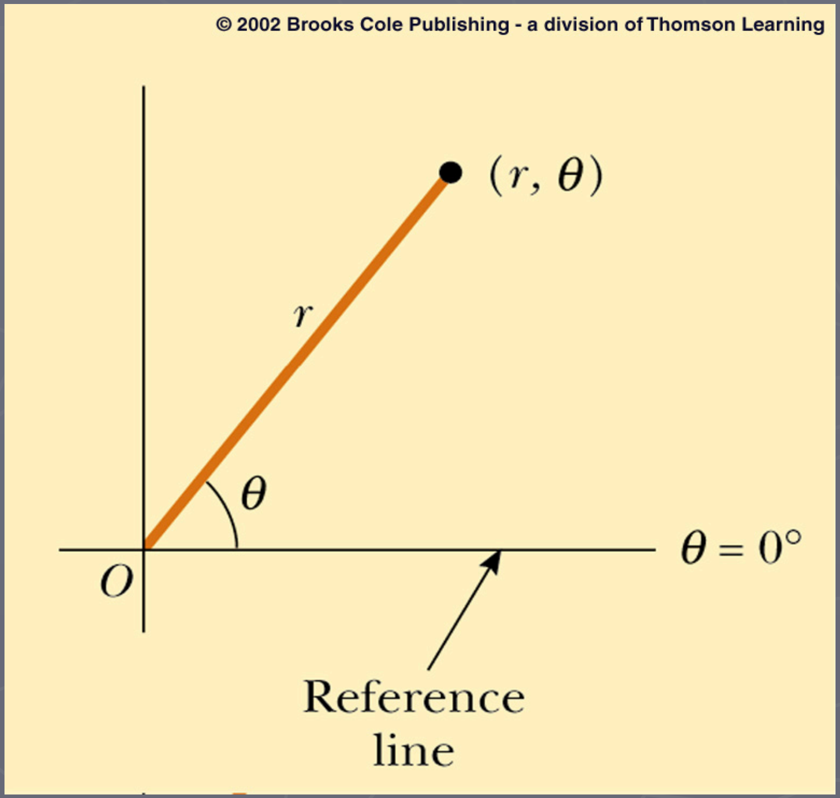
Cartesian coordinate system

- also called rectangular coordinate system
- x- and y- axes
- points are labeled (x,y)



Plane polar coordinate system

- origin and reference line are noted
- point is distance r from the origin in the direction of angle θ, ccw from reference line
- points are labeled (r,θ)

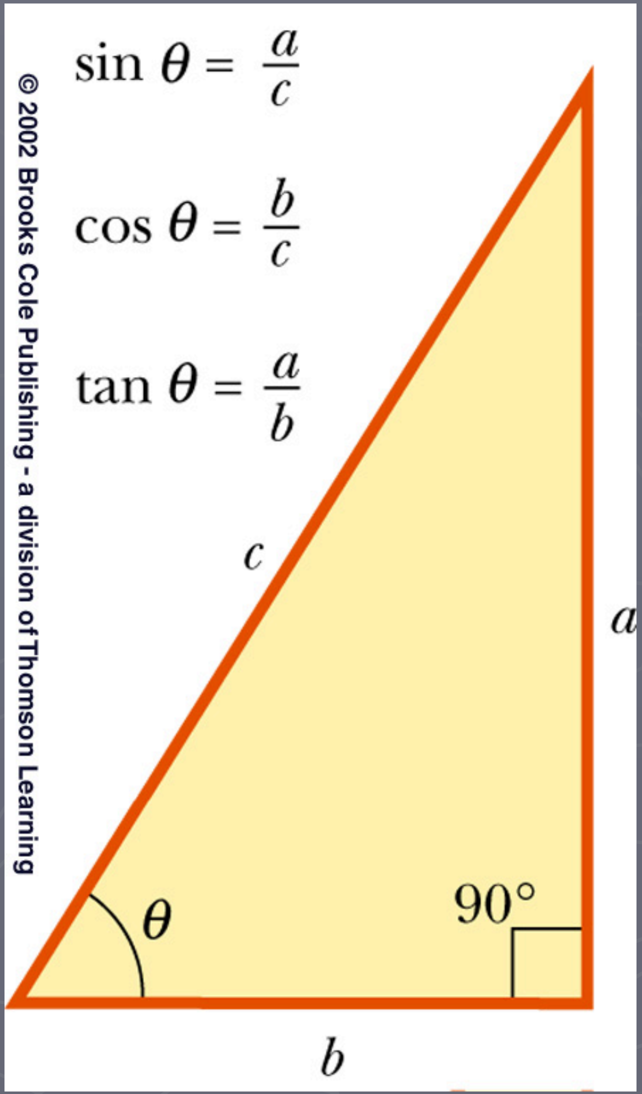


Math Review: Trigonometry

$$\sin\theta = \frac{opposite\ side}{hypotenuse}$$

$$\cos\theta = \frac{adjacent\ side}{hypotenuse}$$

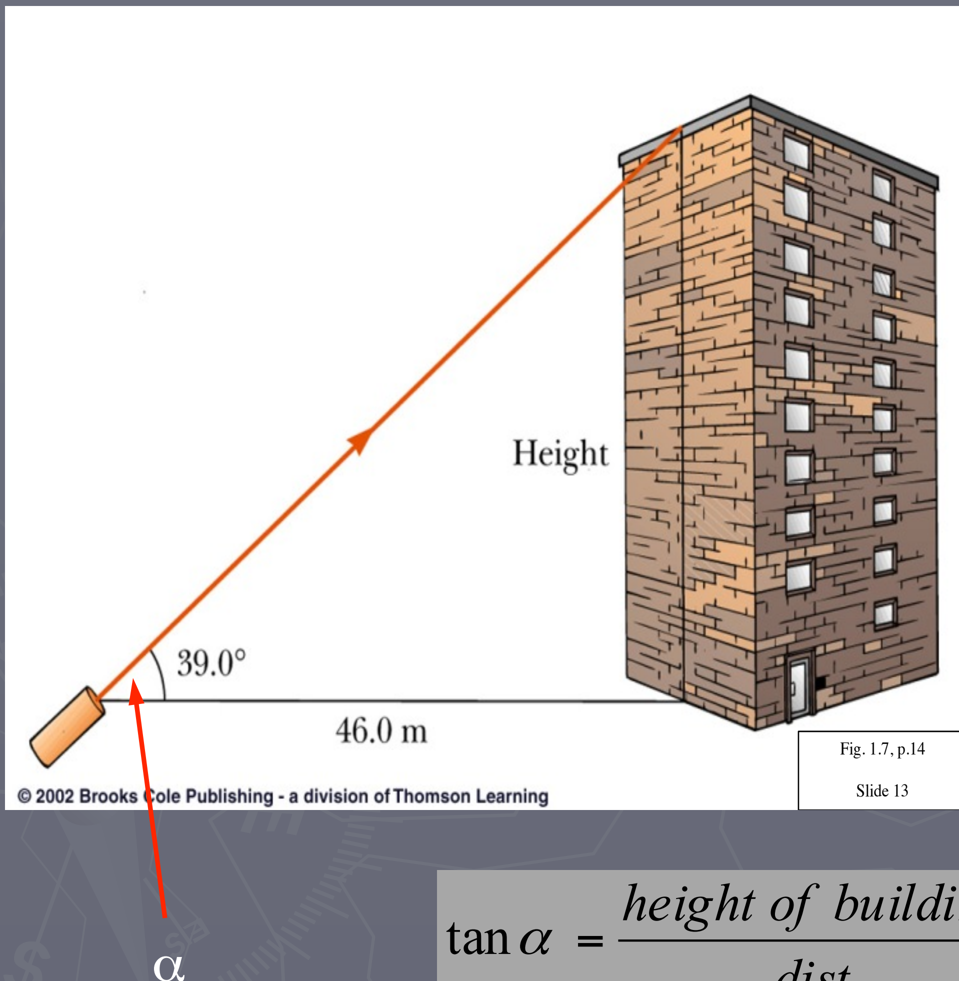
$$\tan\theta = \frac{opposite\ side}{adjacent\ side}$$



- Pythagorean Theorem

$$c^2 = a^2 + b^2$$

Example: how high is the building?



Known: angle and one side
Find: another side

Key: tangent is defined via two sides!

$$\tan\alpha = \frac{height\ of\ building}{dist.},$$

$$height = dist. \times \tan\alpha = (\tan 39.0^\circ)(46.0\ m) = 37.3\ m$$

Math Review: Scalar and Vector Quantities

- Scalar quantities are completely described by magnitude only (temperature, length,…)
- Vector quantities need both magnitude (size) and direction to completely describe them

  (force, displacement, velocity,…)

  - Represented by an arrow, the length of the arrow is proportional to the magnitude of the vector
  - Head of the arrow represents the direction

Vector Notation

- When handwritten, use an arrow: $\vec{A}$
- When printed, will be in bold print: **A**
- When dealing with just the magnitude of a vector in print, an italic letter will be used: *A*

Properties of Vectors

- Equality of Two Vectors
  - Two vectors are **equal** if they have the same magnitude and the same direction
- Movement of vectors in a diagram
  - Any vector can be moved parallel to itself without being affected

More Properties of Vectors

- Negative Vectors
  - Two vectors are **negative** if they have the same magnitude but are 180° apart (opposite directions)
    - **A** = -**B**
- Resultant Vector
  - The **resultant** vector is the sum of a given set of vectors

Adding Vectors

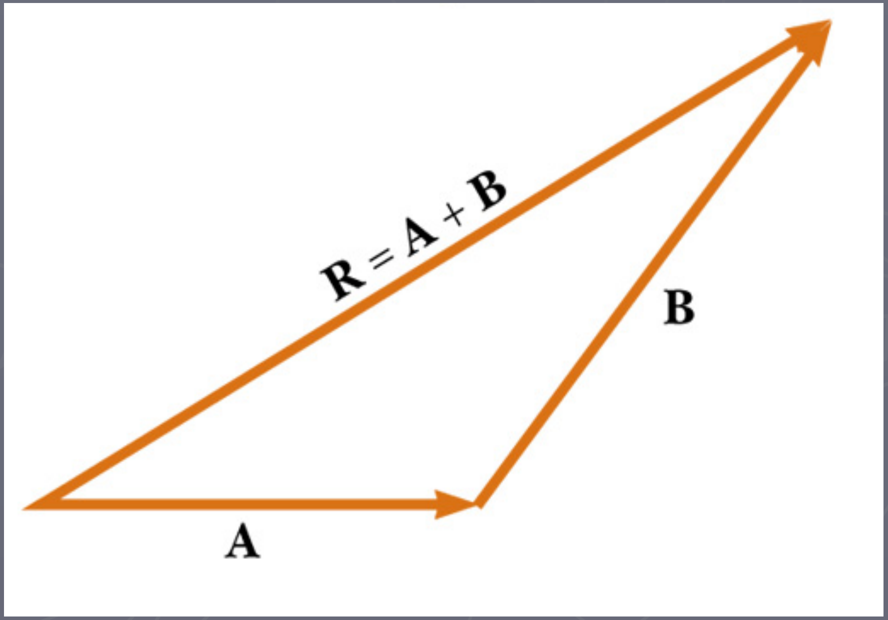
- When adding vectors, their directions must be taken into account
- Units must be the same
- Graphical Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient

Adding Vectors Graphically (Triangle or Polygon Method)

- Choose a scale
- Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector **A** and parallel to the coordinate system used for **A**

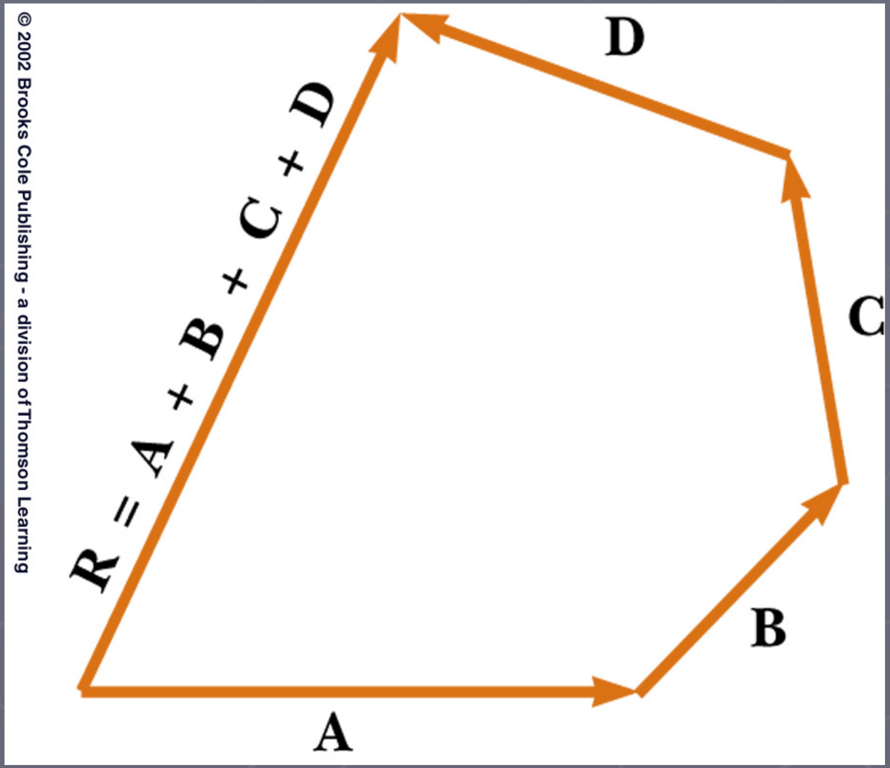
Graphically Adding Vectors

- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of **A** to the end of the last vector
- Measure the length of **R** and its angle
  - Use the scale factor to convert length to actual magnitude



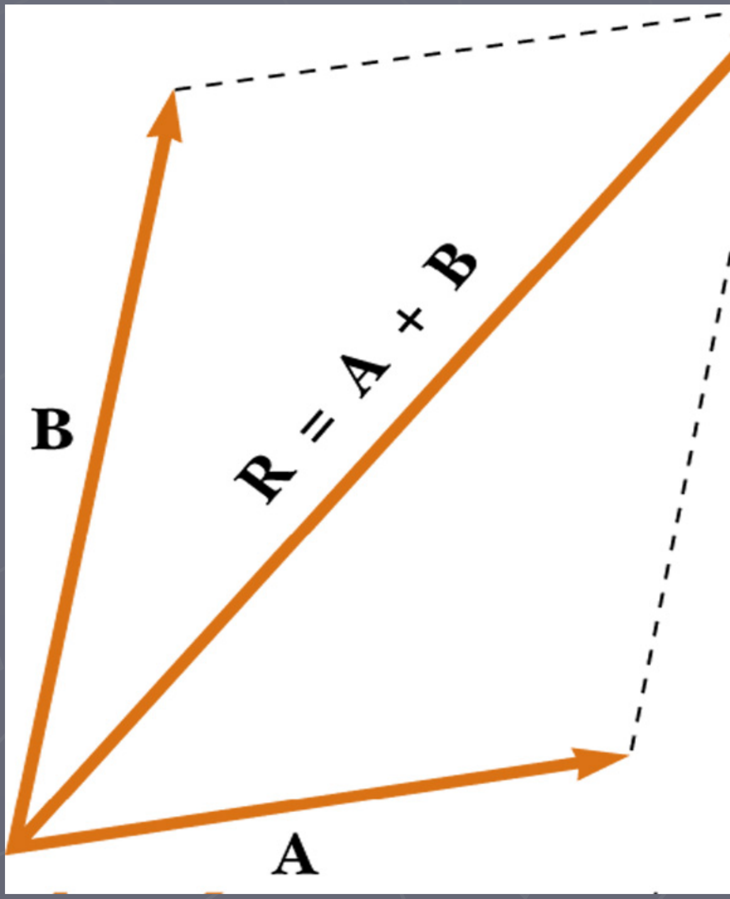
Graphically Adding Vectors

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



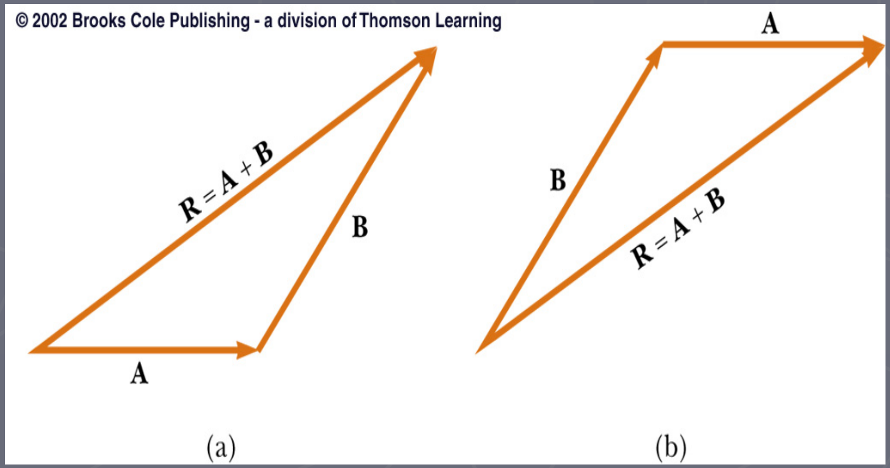
Alternative Graphical Method

- When you have only two vectors, you may use the **Parallelogram Method**
- All vectors, including the resultant, are drawn from a common origin
  - The remaining sides of the parallelogram are sketched to determine the diagonal, **R**



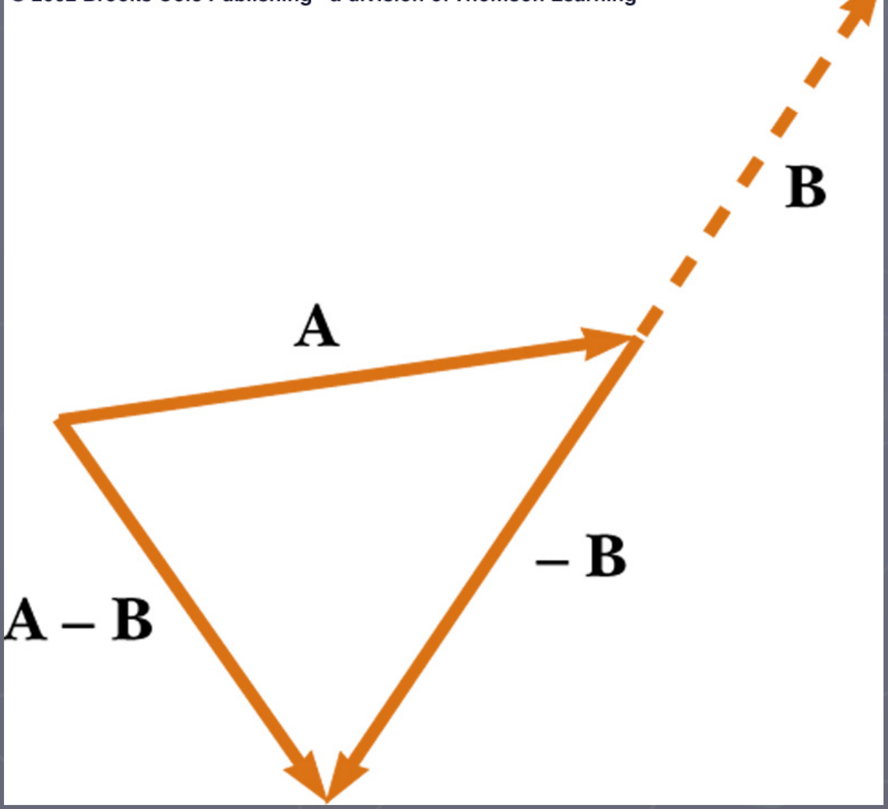
Notes about Vector Addition

- Vectors obey the **Commutative Law of Addition**
  - The order in which the vectors are added doesn't affect the result



Vector Subtraction

- Special case of vector addition
- If **A** – **B**, then use **A**+ (**-B**)
- Continue with standard vector addition procedure



Multiplying or Dividing a Vector by a Scalar

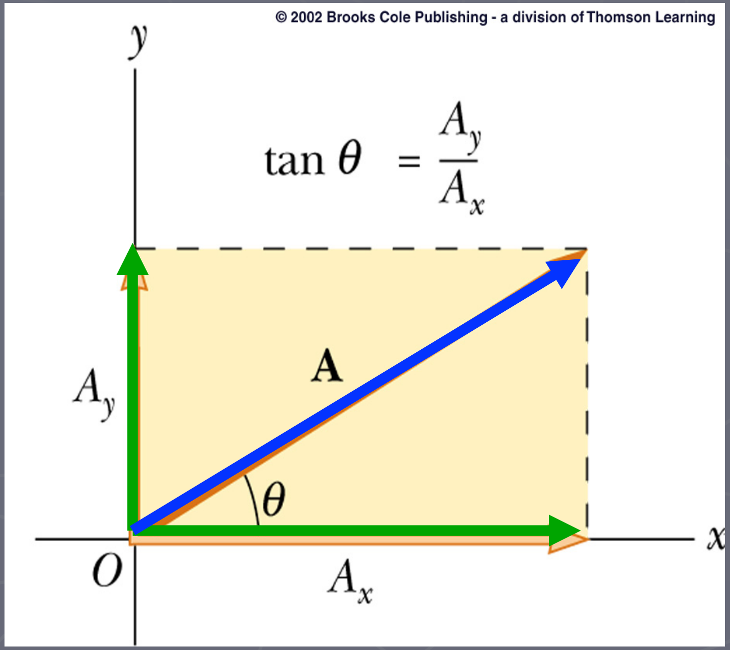
- The result of the multiplication or division is a vector
- The magnitude of the vector is multiplied or divided by the scalar
- If the scalar is positive, the direction of the result is the same as of the original vector
- If the scalar is negative, the direction of the result is opposite that of the original vector

Components of a Vector

Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes
- Vector **A** is now a sum of its components:

$$\boxed{\vec{\mathbf{A}} = \vec{A}_x + \vec{A}_y}$$



What are $\vec{A}_x$ and $\vec{A}_y$?

Components of a Vector

- The components are the legs of the right triangle whose hypotenuse is **A**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

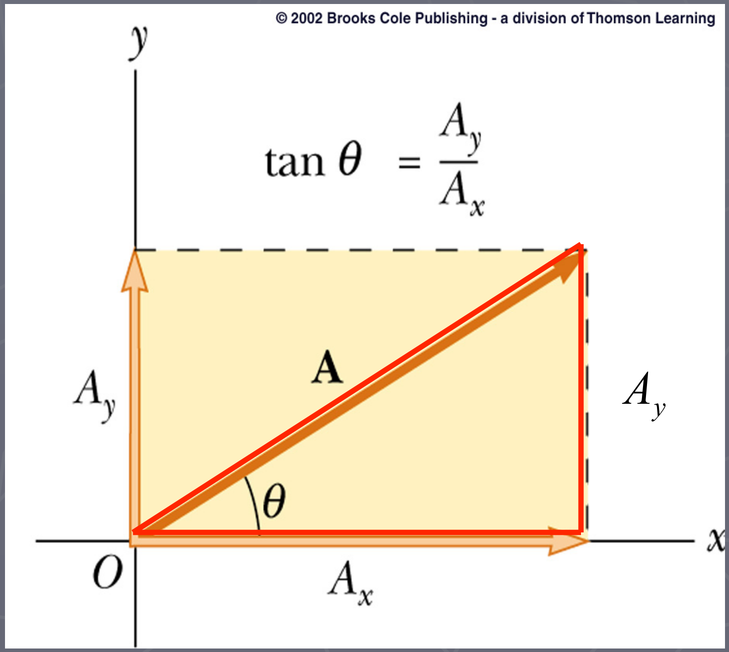
- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

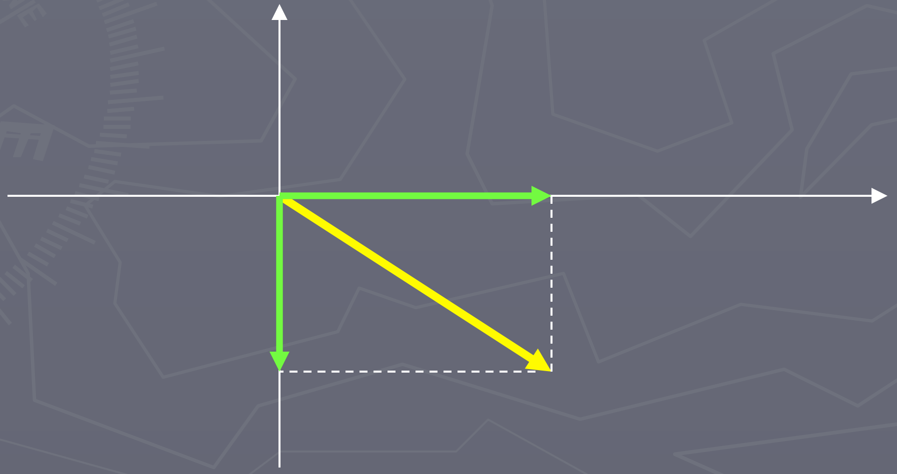
$$A_y = A\sin\theta$$

- Then, $\boxed{\vec{\mathbf{A}} = \vec{A}_x + \vec{A}_y}$



Notes About Components

- The previous equations are valid ***only if θ is measured with respect to the x-axis***
- The components can be positive or negative and will have the same units as the original vector



Example 1

A golfer takes two putts to get his ball into the hole once he is on the green. The first putt displaces the ball 6.00 m east, and the second, 5.40 m south. What displacement would have been needed to get the ball into the hole on the first putt?

Given:

Δx_1= 6.00 m (east)
Δx_2= 5.40 m (south)

Find:

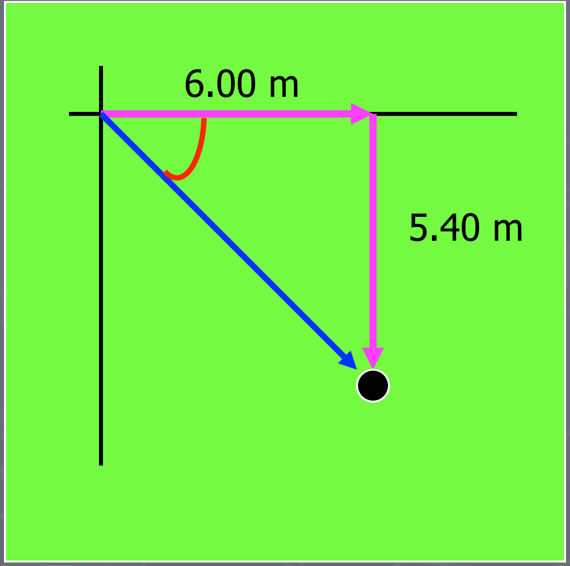
R = ?

Solution:

1. Note right triangle, use Pythagorean theorem

$$R = \sqrt{(6.00\text{ m})^2 + (5.40\text{ m})^2} = 8.07\text{ m}$$

2. Find angle:



$$\theta = \tan^{-1}\left(\frac{5.40\text{ m}}{6.00\text{ m}}\right) = \tan^{-1}(0.900) = 42.0^\circ$$

What Components Are Good For: Adding Vectors Algebraically

- Choose a coordinate system and sketch the vectors v_1, v_2, …
- Find the x- and y-components of all the vectors
- Add all the x-components
  - This gives R_x:

$$R_x = \sum v_x$$

- Add all the y-components
  - This gives R_y:

$$R_y = \sum v_y$$

Magnitudes of vectors pointing in the same direction can be added to find the resultant!

Adding Vectors Algebraically (cont.)

- Use the Pythagorean Theorem to find the magnitude of the Resultant:

$$R = \sqrt{R_x^2 + R_y^2}$$

- Use the inverse tangent function to find the direction of R:

$$\theta = \tan^{-1}\frac{R_y}{R_x}$$

Example:

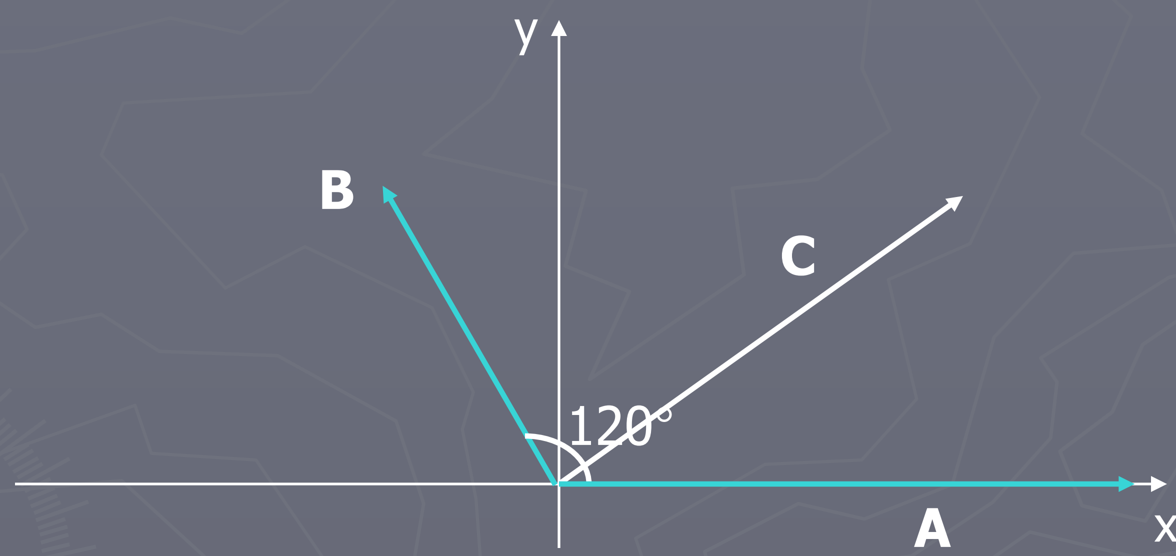
A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, then 6.00 blocks east. How far did she move from her original position?

§3.1 Graphical Addition and Subtraction of Vectors

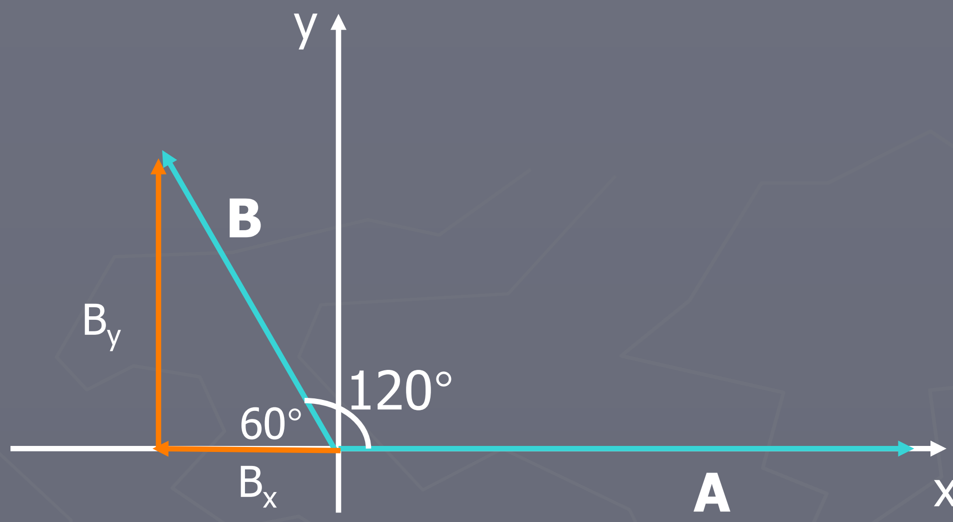
A **vector** is a quantity that has both a **magnitude** and a **direction**. Position is an example of a vector quantity.

A **scalar** is a quantity with no direction. The mass of an object is an example of a scalar quantity.

Example: Vector **A** has a length of 5.00 meters and points along the x-axis. Vector **B** has a length of 3.00 meters and points 120° from the +x-axis. Compute **A**+**B** (=**C**).



Example continued:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opp}}{\text{adj}}$$

$$\sin 60° = \frac{B_y}{B} \Rightarrow B_y = B\sin 60° = (3.00\,\text{m})\sin 60° = 2.60\ \text{m}$$

$$\cos 60° = \frac{-B_x}{B} \Rightarrow B_x = -B\cos 60° = -(3.00\,\text{m})\cos 60° = -1.50\ \text{m}$$

and A_x = 5.00 m and A_y = 0.00 m