General Physics (PHY 2130)

Lecture 6

- Vectors (cont.)
- Motion in two dimensions
  - projectile motion

http://www.physics.wayne.edu/~apetrov/PHY2130/

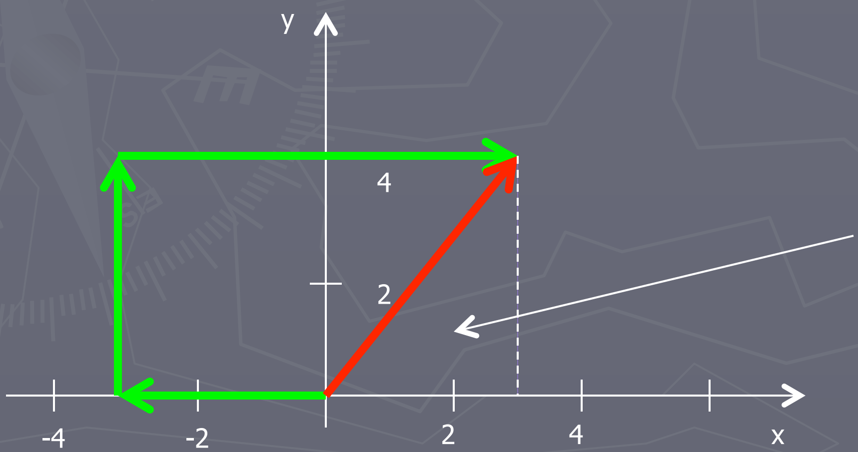
WAYNE STATE UNIVERSITY

Lightning Review

Last lecture:

1. Vectors: objects that need both magnitude and direction to define them
   - ✓ coordinate systems (frames): cartesian and polar
   - ✓ addition and subtraction of vectors, other operations

Review Problem: A girl delivering newspapers covers her route by traveling 3.00 blocks west, 4.00 blocks north, then 6.00 blocks east. How far did she move from her original position?



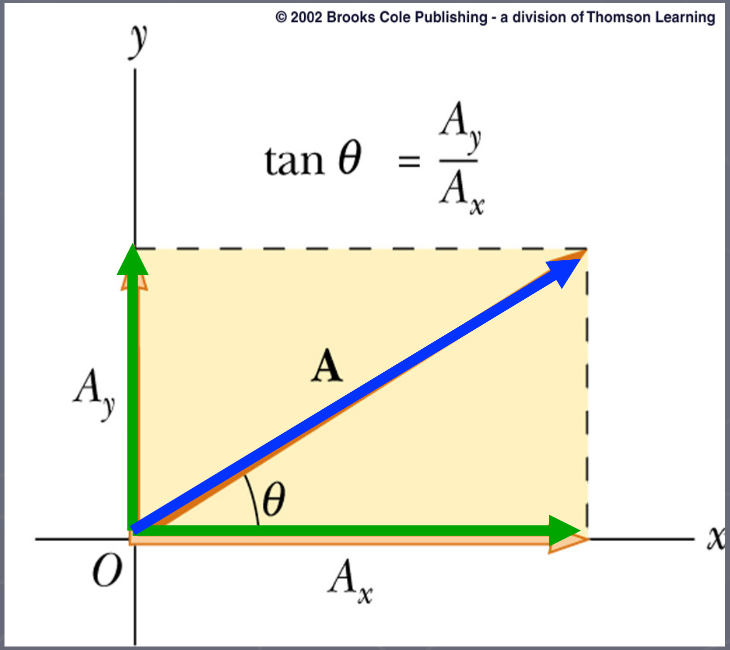
From this triangle:

$$R = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(3\,bl)^2 + (4\,bl)^2}$$

$$= 5\ blocks$$

Recall: Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components**
  - These are the projections of the vector along the x- and y-axes
- Vector **A** is now a sum of its components:



$$\boxed{\vec{\mathbf{A}} = \vec{A}_x + \vec{A}_y}$$

What are $\vec{A}_x$ and $\vec{A}_y$?

Recall: Components of a Vector

- The components are the legs of the right triangle whose hypotenuse is **A**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

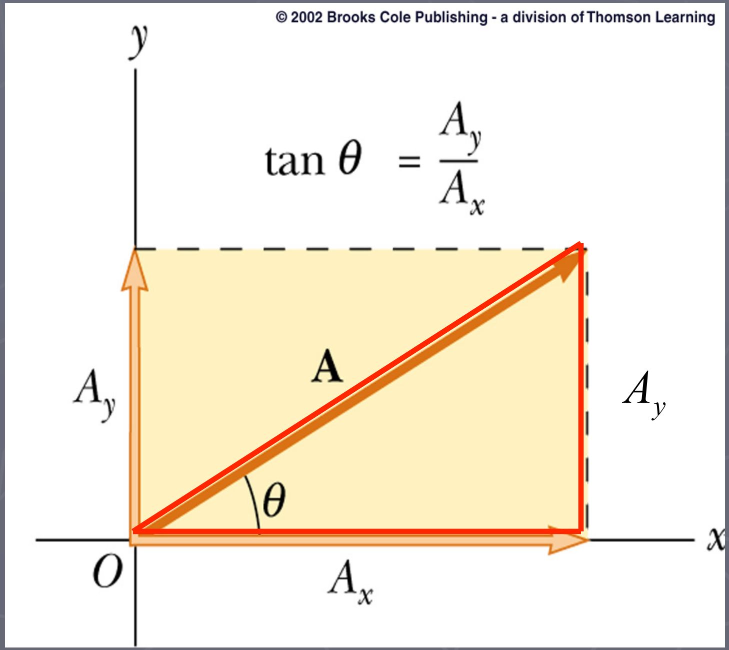
- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

$$A_y = A\sin\theta$$

- Then, $\boxed{\vec{\mathbf{A}} = \vec{A}_x + \vec{A}_y}$



What Components Are Good For: Adding Vectors Algebraically

- Choose a coordinate system and sketch the vectors v_1, v_2, …
- Find the x- and y-components of all the vectors
- Add all the x-components
  - This gives R_x:

$$R_x = \sum v_x$$

- Add all the y-components
  - This gives R_y:

$$R_y = \sum v_y$$

Magnitudes of vectors pointing in the same direction can be added to find the resultant!

Adding Vectors Algebraically (cont.)

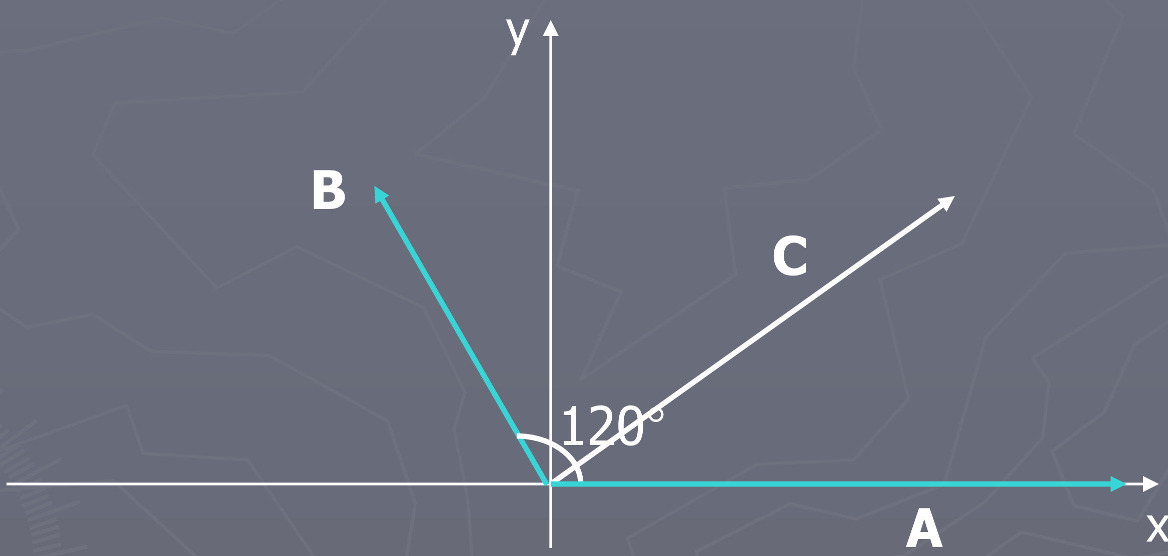
- Use the Pythagorean Theorem to find the magnitude of the Resultant:

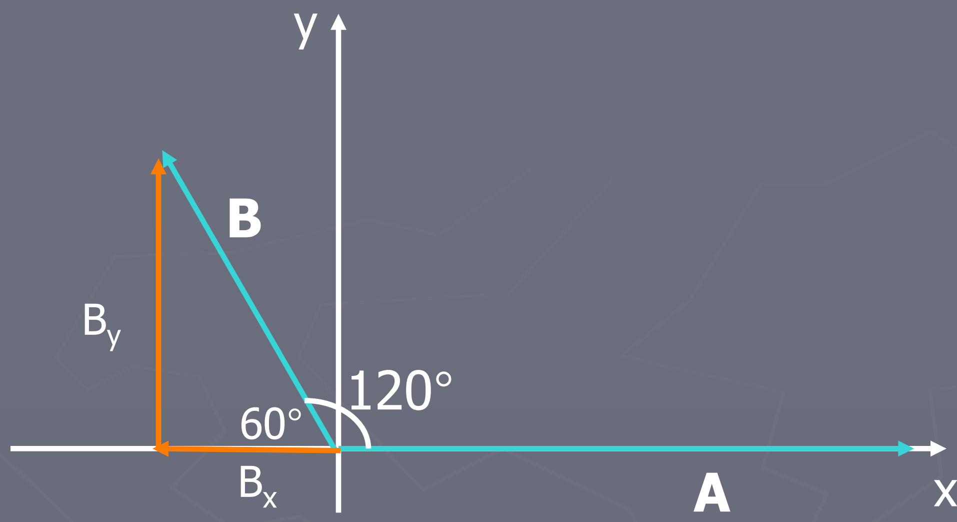
$$R = \sqrt{R_x^2 + R_y^2}$$

- Use the inverse tangent function to find the direction of R:

$$\theta = \tan^{-1} \frac{R_y}{R_x}$$

Example: Vector **A** has a length of 5.00 meters and points along the x-axis. Vector **B** has a length of 3.00 meters and points 120° from the +x-axis. Compute **A**+**B** (=**C**).





$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{opp}}{\text{adj}}$$

$$\sin 60^\circ = \frac{B_y}{B} \Rightarrow B_y = B\sin 60^\circ = (3.00\,\text{m})\sin 60^\circ = 2.60\ \text{m}$$

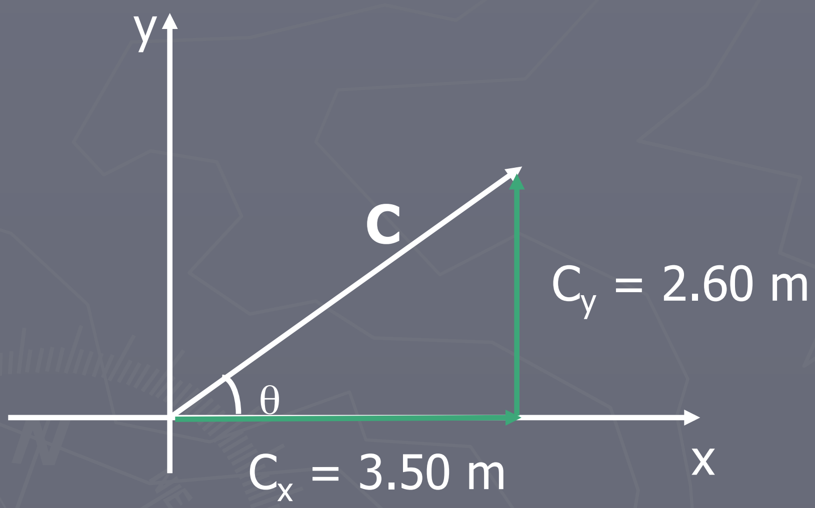
$$\cos 60^\circ = \frac{-B_x}{B} \Rightarrow B_x = -B\cos 60^\circ = -(3.00\,\text{m})\cos 60^\circ = -1.50\ \text{m}$$

and A_x = 5.00 m and A_y = 0.00 m

The components of **C**:

$$C_x = A_x + B_x = 5.00\ \text{m} + (-1.50\ \text{m}) = 3.50\ \text{m}$$

$$C_y = A_y + B_y = 0.00\ \text{m} + 2.60\ \text{m} = 2.60\ \text{m}$$



The length of C is:

$$C = |\mathbf{C}| = \sqrt{C_x^{\ 2} + C_y^{\ 2}}$$

$$= \sqrt{(3.50\ \text{m})^2 + (2.60\ \text{m})^2}$$

$$= 4.36\ \text{m}$$

The direction of C is:

$$\tan\theta = \frac{C_y}{C_x} = \frac{2.60\ \text{m}}{3.50\ \text{m}} = 0.7429$$

$$\theta = \tan^{-1}(0.7429) = 36.6^\circ$$ From the +x-axis

Motion in Two Dimensions

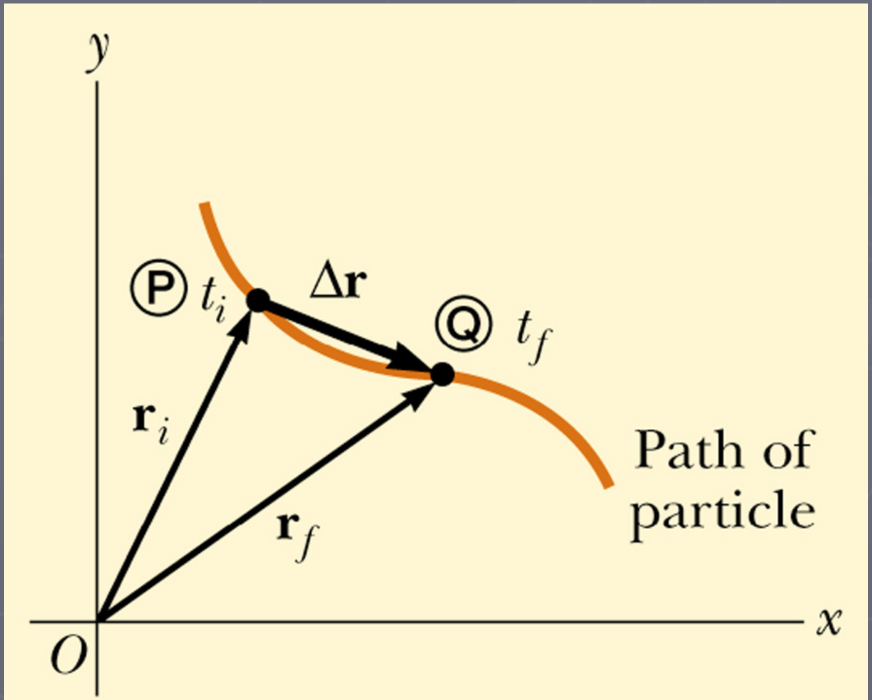
Motion in Two Dimensions

- Using + or – signs is not always sufficient to fully describe motion in more than one dimension
  - Vectors can be used to more fully describe motion
- Still interested in displacement, velocity, and acceleration

Displacement

- The position of an object is described by its position vector, **r**
- The **displacement** of the object is defined as the ***change in its position***

$$\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$$



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Velocity

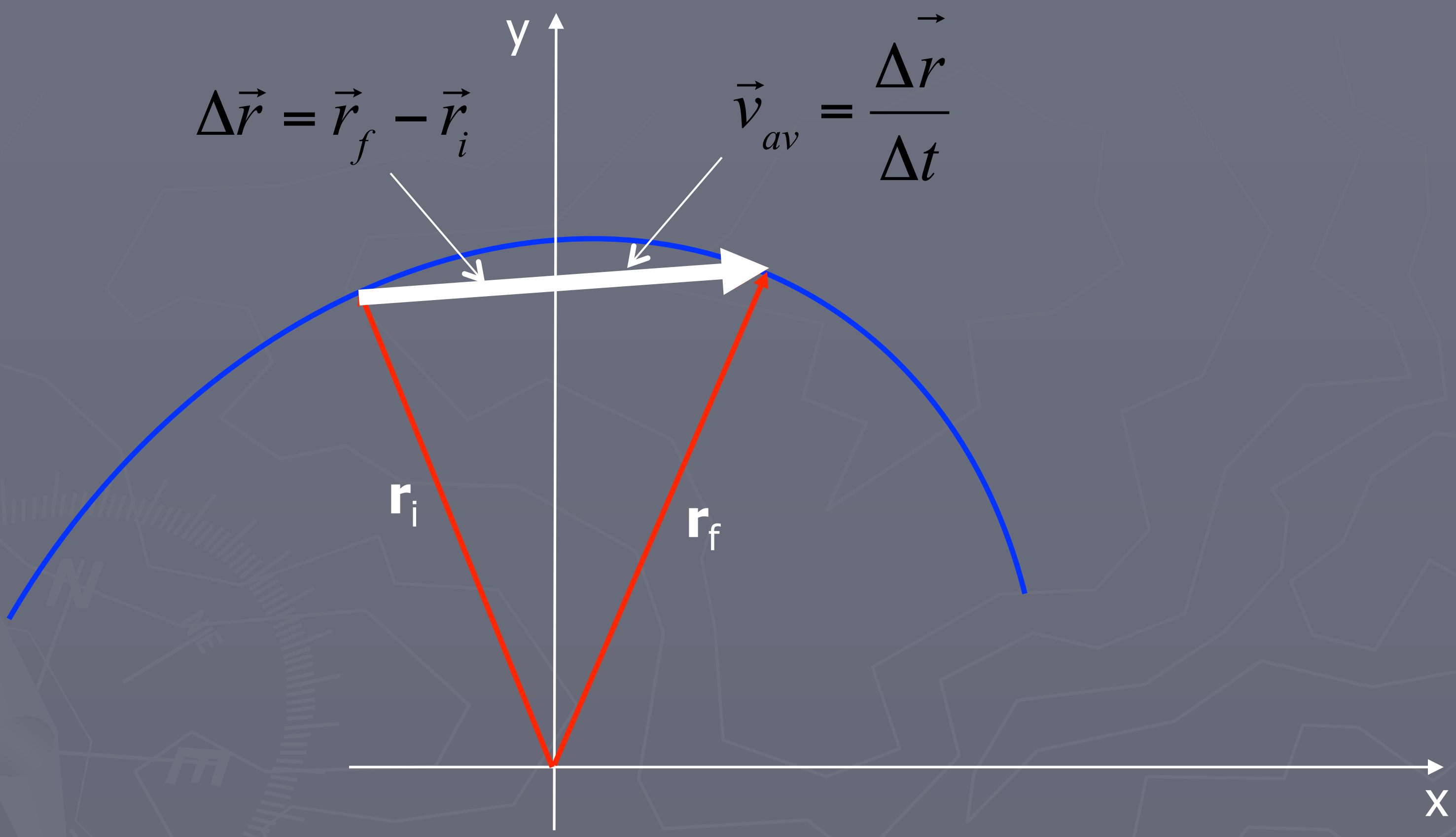
- The average velocity is the ratio of the displacement to the time interval for the displacement

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

- The instantaneous velocity is the limit of the average velocity as Δt approaches zero
  - The direction of the instantaneous velocity is along a line that is tangent to the path of the particle and in the direction of motion

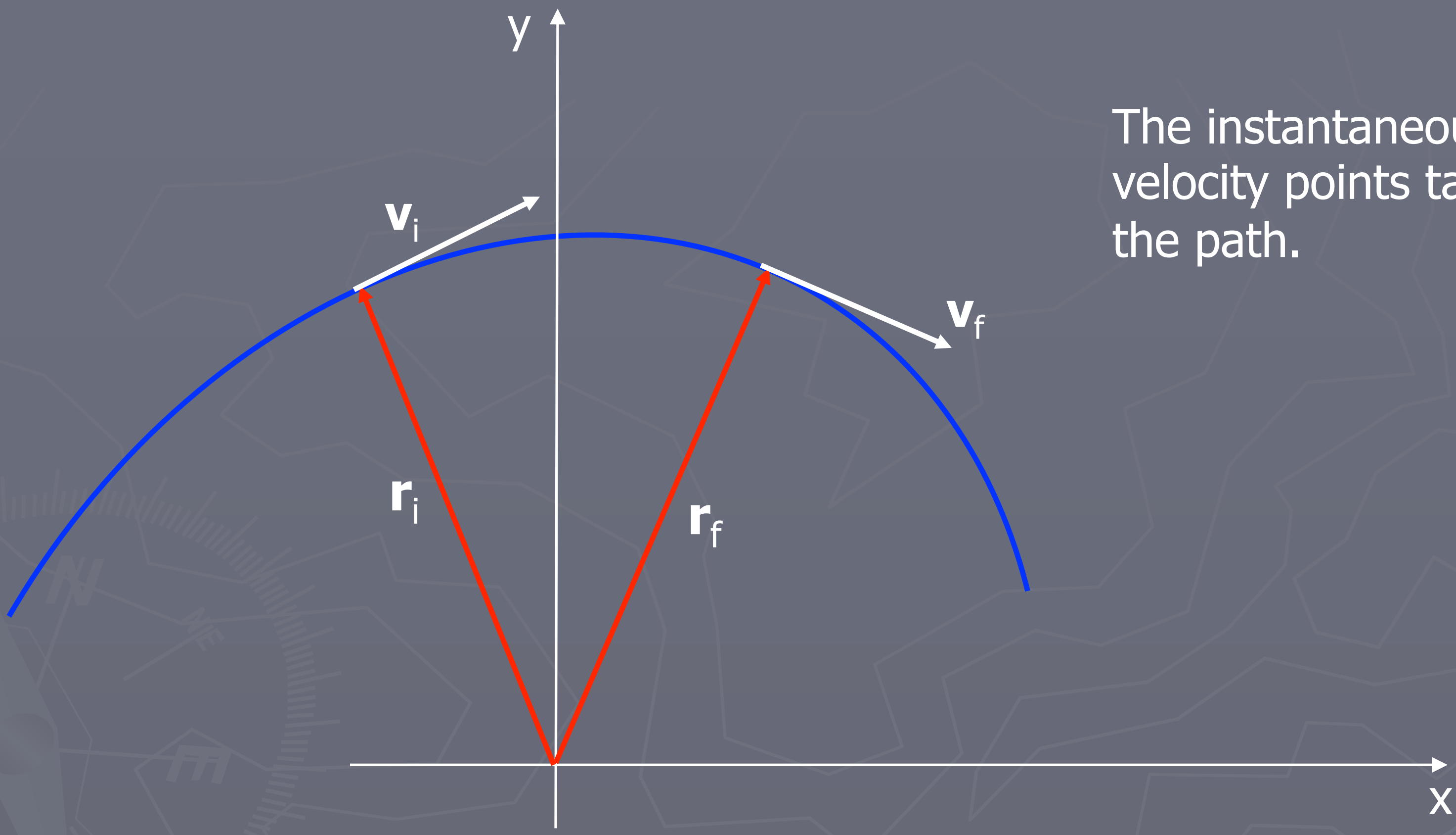
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

A particle moves along the blue path as shown. At time t_1 its position is $\mathbf{r}_i$ and at time t_2 its position is $\mathbf{r}_f$.



Average velocity is directed along the displacement!

The instantaneous velocity:



The instantaneous velocity points tangent to the path.

Acceleration

- The average acceleration is defined as the rate at which the velocity changes

$$\bar{a} = \frac{\Delta \vec{v}}{\Delta t}$$

- The instantaneous acceleration is the limit of the average acceleration as Δ t approaches zero

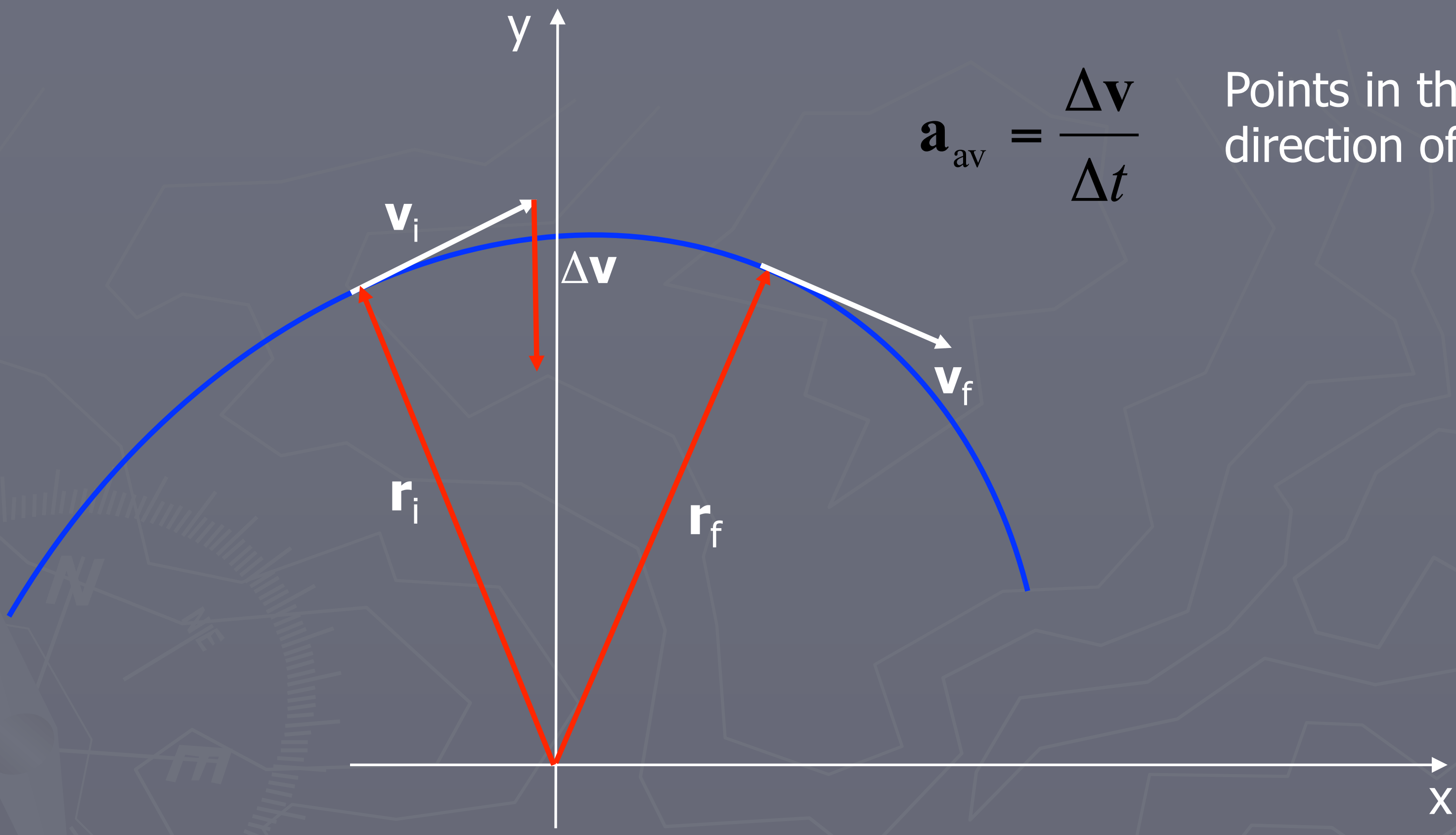
$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Ways an Object Might Accelerate

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

- The magnitude of the velocity (the speed) can change
- The direction of the velocity can change
  - Even though the magnitude is constant
- Both the magnitude and the direction can change

A particle moves along the blue path as shown. At time t_1 its position is $\mathbf{r}_0$ and at time t_2 its position is $\mathbf{r}_f$.



$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t}$$

Points in the direction of $\Delta\mathbf{v}$.

The instantaneous acceleration can point in any direction.

Big Example: Projectile Motion

- An object may move in both the x and y directions simultaneously (i.e. in two dimensions)
- The form of two dimensional motion we will deal with is called **projectile motion**
- We may:
  - ignore air friction
  - ignore the rotation of the earth
- With these assumptions, an object in projectile motion will follow a parabolic path

Notes on Projectile Motion:

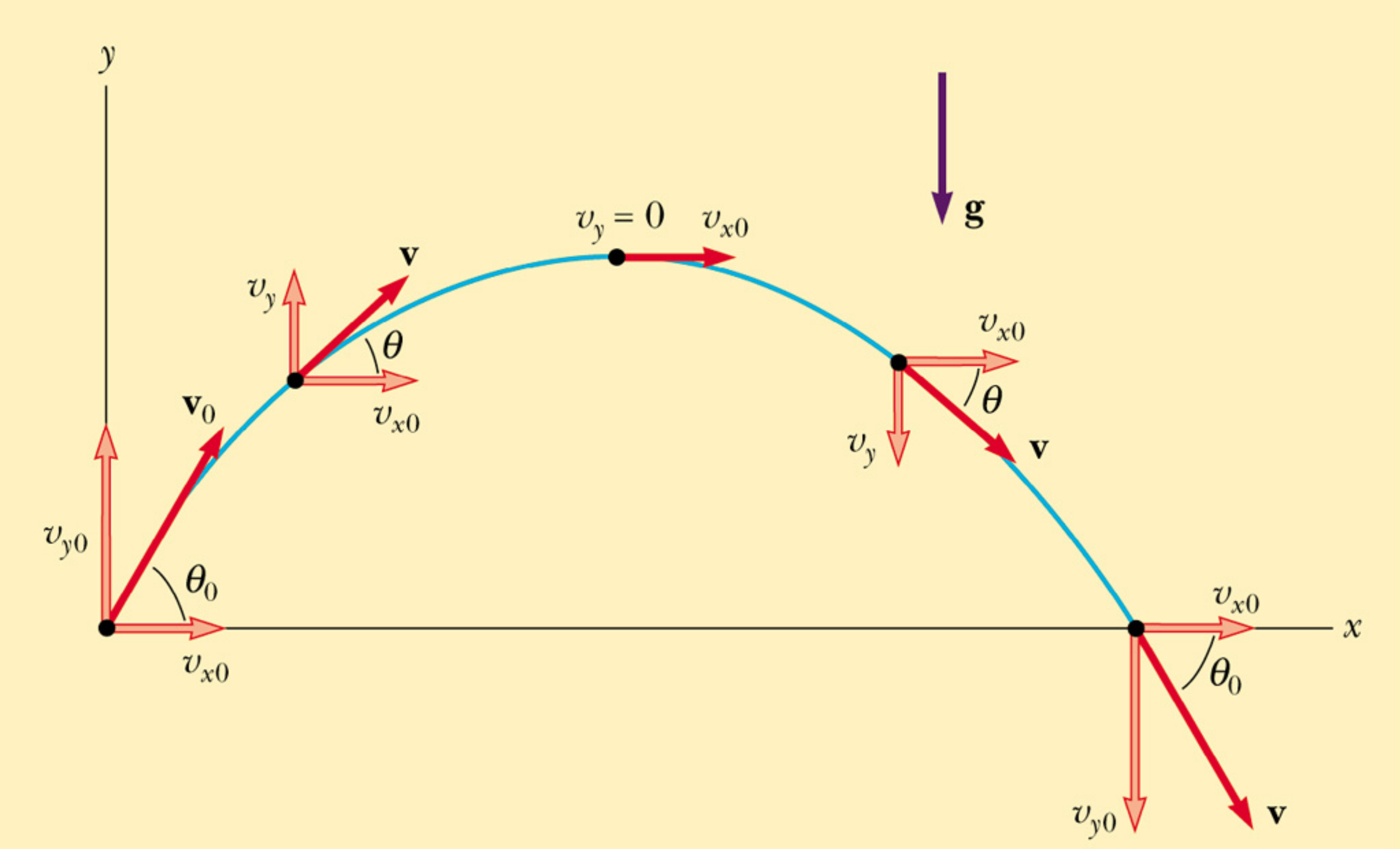
- once released, only gravity pulls on the object, just like in up-and-down motion

- since gravity pulls on the object downwards:

  ✓ vertical acceleration downwards

  ✓ NO acceleration in horizontal direction

Projectile Motion



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Rules of Projectile Motion

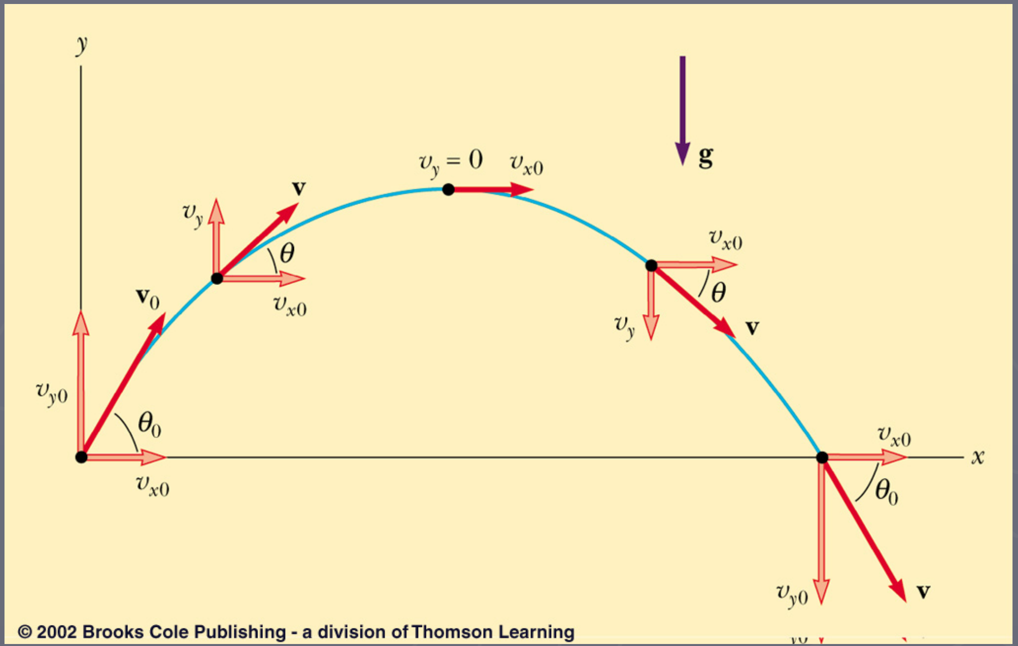
- Introduce coordinate frame: y is up
- The x- and y-components of motion can be treated independently
- Velocities (incl. initial velocity) can be broken down into its x- and y-components
- The x-direction is uniform motion

$$a_x = 0$$

- The y-direction is free fall

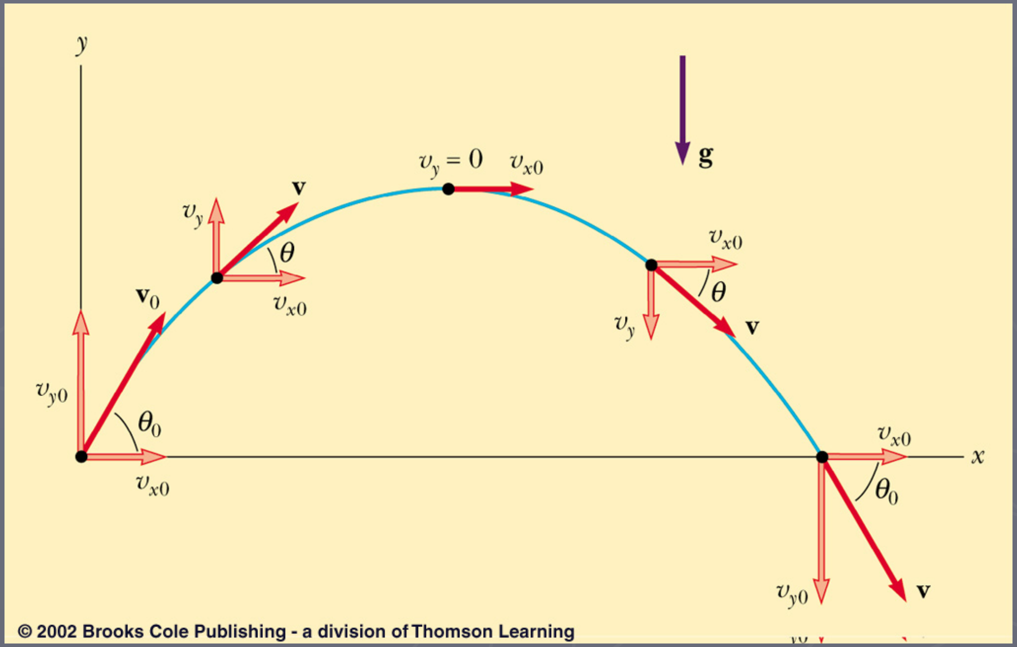
$$|a_y| = g$$

Some Details About the Rules



- x-direction
  - $a_x = 0$
  - $v_{xo} = v_o \cos\theta_o = v_x = \text{constant}$
  - $x = v_{xo}t$
    - This is the only operative equation in the x-direction since there is uniform velocity in that direction

More Details About the Rules



- y-direction
  - $v_{yo} = v_o \sin\theta_o$
  - take the positive direction as upward
  - then: free fall problem
    - only then: a_y = -g (in general, $|a_y|$= g)
  - uniformly accelerated motion, so the motion equations all hold

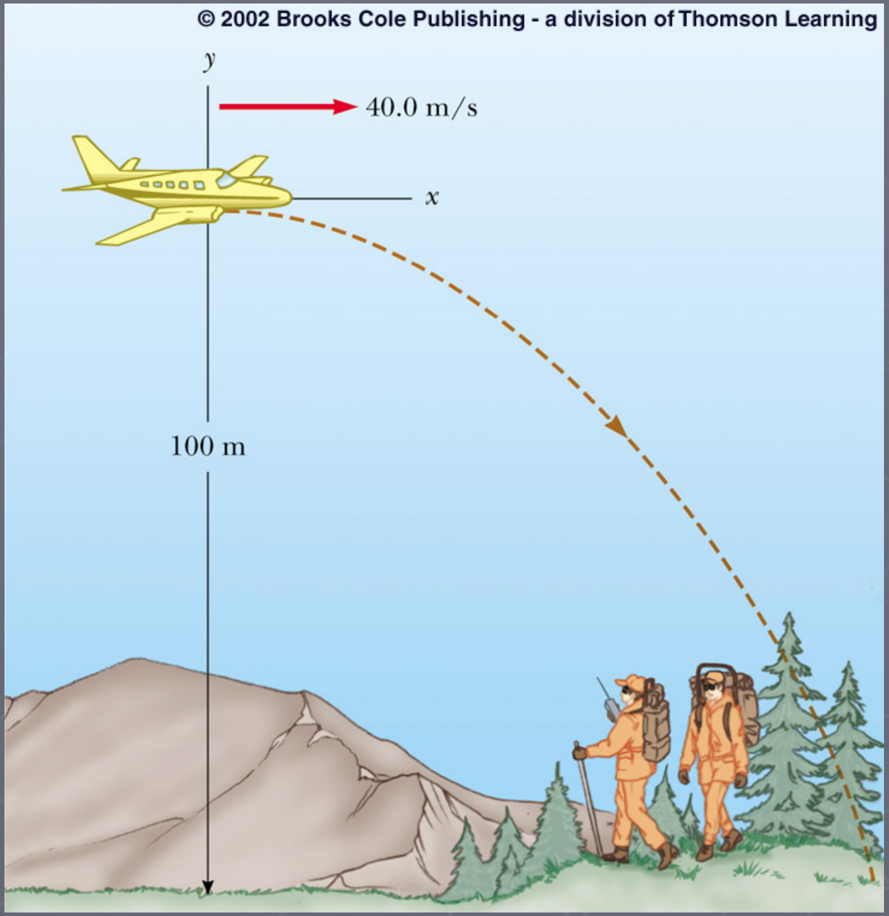
Velocity of the Projectile

- The velocity of the projectile at any point of its motion is the vector sum of its x and y components at that point

$$\boxed{v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{v_y}{v_x}}$$

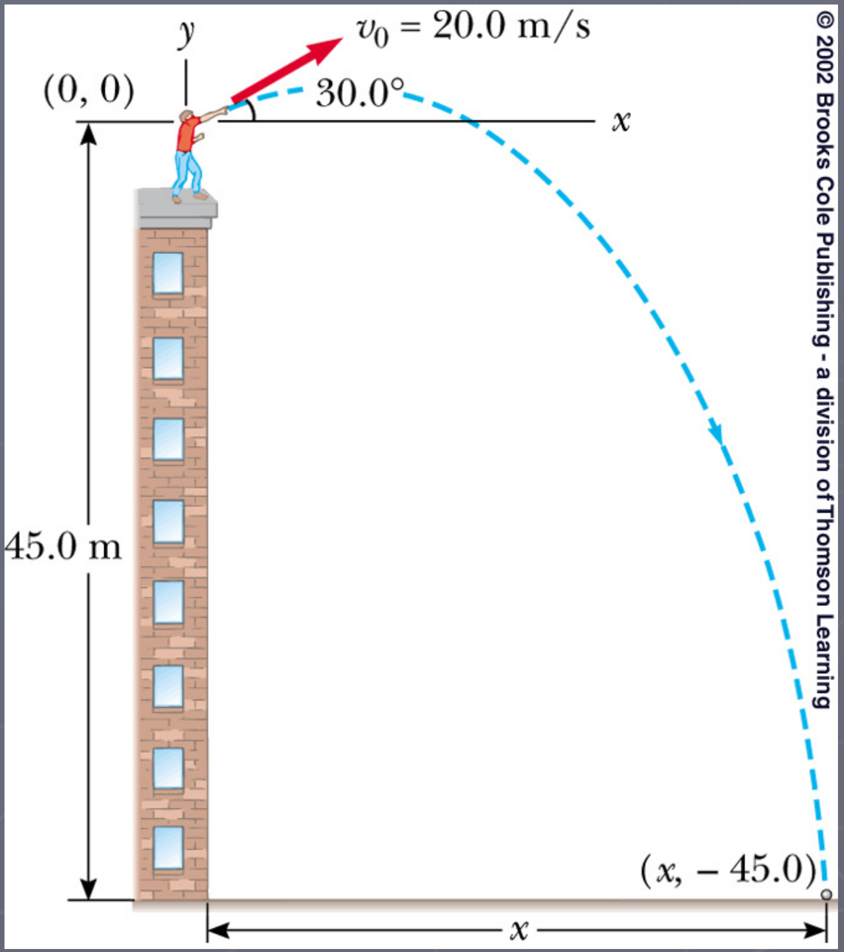
Examples of Projectile Motion:

- An object may be fired horizontally
- The initial velocity is all in the x-direction
  - $v_o = v_x$ and $v_y = 0$
- All the general rules of projectile motion apply



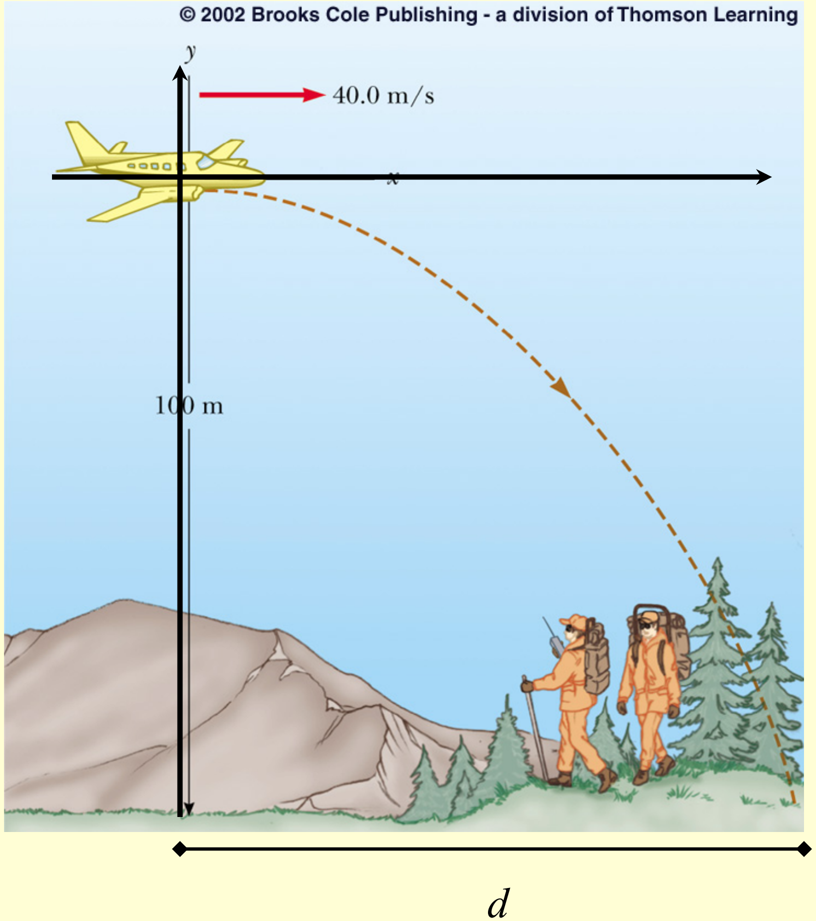
Non-Symmetrical Projectile Motion

- Follow the general rules for projectile motion
- Break the y-direction into parts
  - up and down
  - symmetrical back to initial height and then the rest of the height



Example problem:

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers. The plane is traveling horizontally at 40.0 m/s at a height of 100 m above the ground.
Where does the package strike the ground relative to the point at which it was released?



Given:

velocity: v=40.0 m/s
height: h=100 m

Find:

Distance d=?

1. Introduce coordinate frame:
   Oy: y is directed up
   Ox: x is directed right

2. Note: $v_{ox} = v = +40$ m/s
   $v_{oy} = 0$ m/s

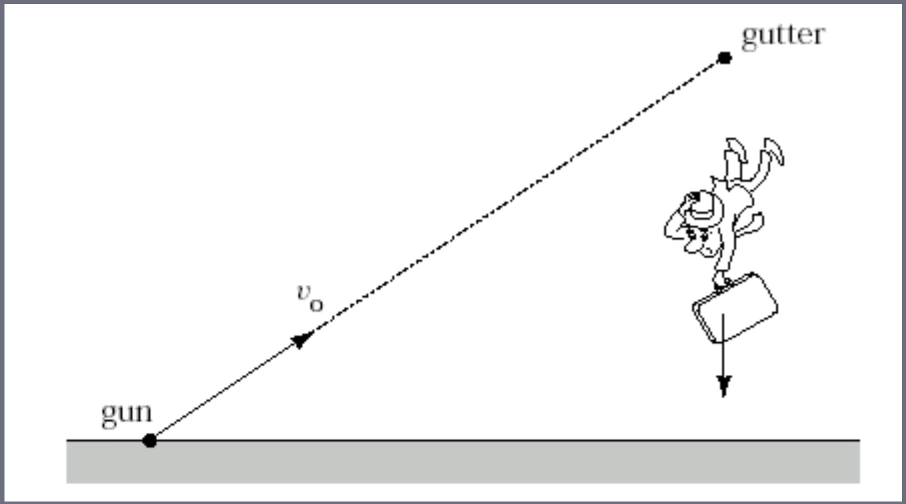
$$\underline{Oy}: y = \frac{1}{2}gt^2, so \quad t = \sqrt{\frac{2y}{g}}$$

$$or: \quad t = \sqrt{\frac{2(-100\,m)}{-9.8\,m/s^2}} = 4.51s$$

$$\underline{Ox}: x = v_{x0}t, so \quad x = (40\,m/\cancel{s})(4.51\cancel{s}) = \underline{180\,m}$$ ✓

What do you think?

Consider the situation depicted here. A gun is accurately aimed at a dangerous criminal hanging from the gutter of a building. The target is well within the gun's range, but the instant the gun is fired and the bullet moves with a speed v_o, the criminal lets go and drops to the ground. What happens? The bullet



1. hits the criminal regardless of the value of v_o.
2. hits the criminal only if v_o is large enough.
3. misses the criminal.

What do you think?

Consider the situation depicted here. A gun is accurately aimed at a dangerous criminal hanging from the gutter of a building. The target is well within the gun's range, but the instant the gun is fired and the bullet moves with a speed v_o, the criminal lets go and drops to the ground. What happens? The bullet

1. hits the criminal regardless of the value of v_o. ✓
2. hits the criminal only if v_o is large enough.
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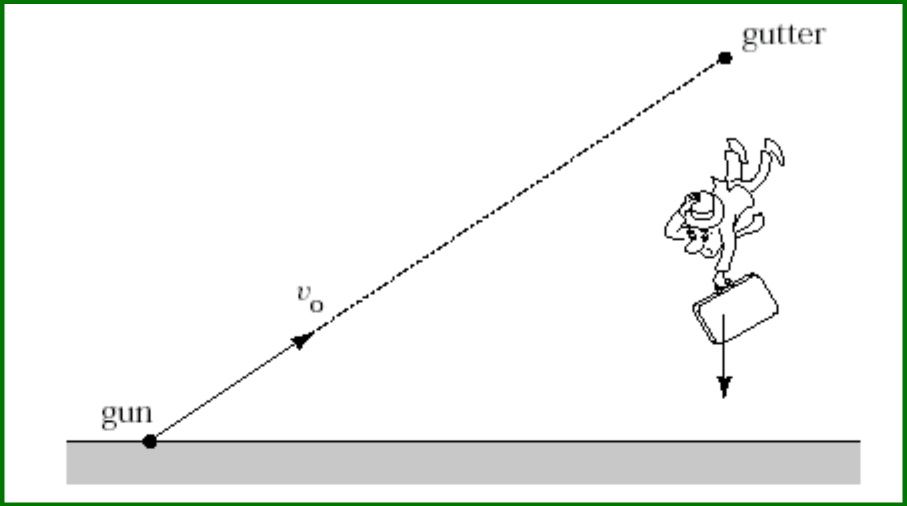
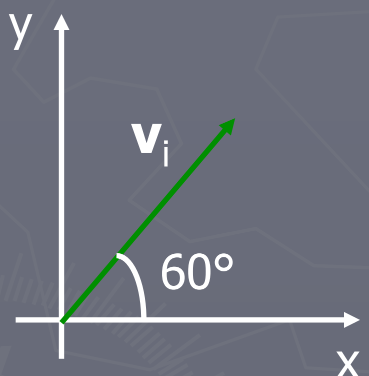


Note: The downward acceleration of the bullet and the criminal are identical, so the bullet will hit the target – they both "fall" the same distance!

Example: An arrow is shot into the air with θ = 60° and v_i = 20.0 m/s.

(a) What are v_x and v_y of the arrow when t = 3 sec?



The components of the initial velocity are:

$$v_{ix} = v_i \cos\theta = 10.0\ \text{m/s}$$

$$v_{iy} = v_i \sin\theta = 17.3\ \text{m/s}$$

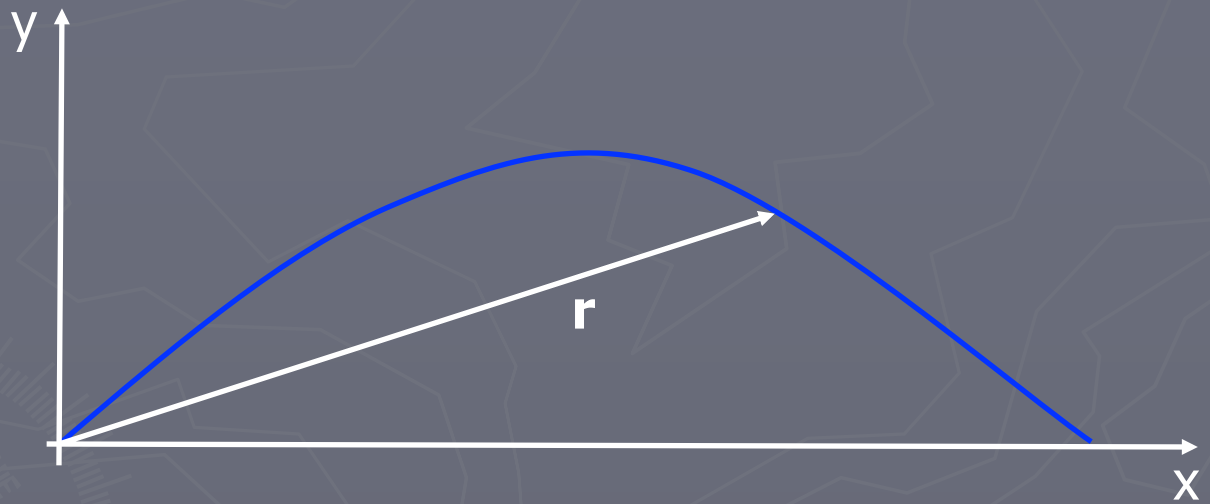
At t = 3 sec:

$$v_{fx} = v_{ix} + a_x \Delta t = v_{ix} = 10.0\ \text{m/s}$$

$$v_{fy} = v_{iy} + a_y \Delta t = v_{iy} - g\Delta t = -12.1\ \text{m/s}$$

Example continued:

(b) What are the x and y components of the displacement of the arrow during the 3.0 sec interval?



$$\Delta r_x = \Delta x = v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2 = v_{ix}\Delta t + 0 = 30.0\,\mathrm{m}$$

$$\Delta r_y = \Delta y = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2 = v_{iy}\Delta t - \frac{1}{2}g\Delta t^2 = 7.80\,\mathrm{m}$$

Example: How far does the arrow in the previous example land from where it is released?

The arrow lands when $\Delta y = 0$.

$$\Delta y = v_{iy}\Delta t - \frac{1}{2}g\Delta t^2 = 0$$

Solving for Δt:

$$\Delta t = \frac{2v_{iy}}{g} = 3.53\ \text{sec}$$

The distance traveled is:

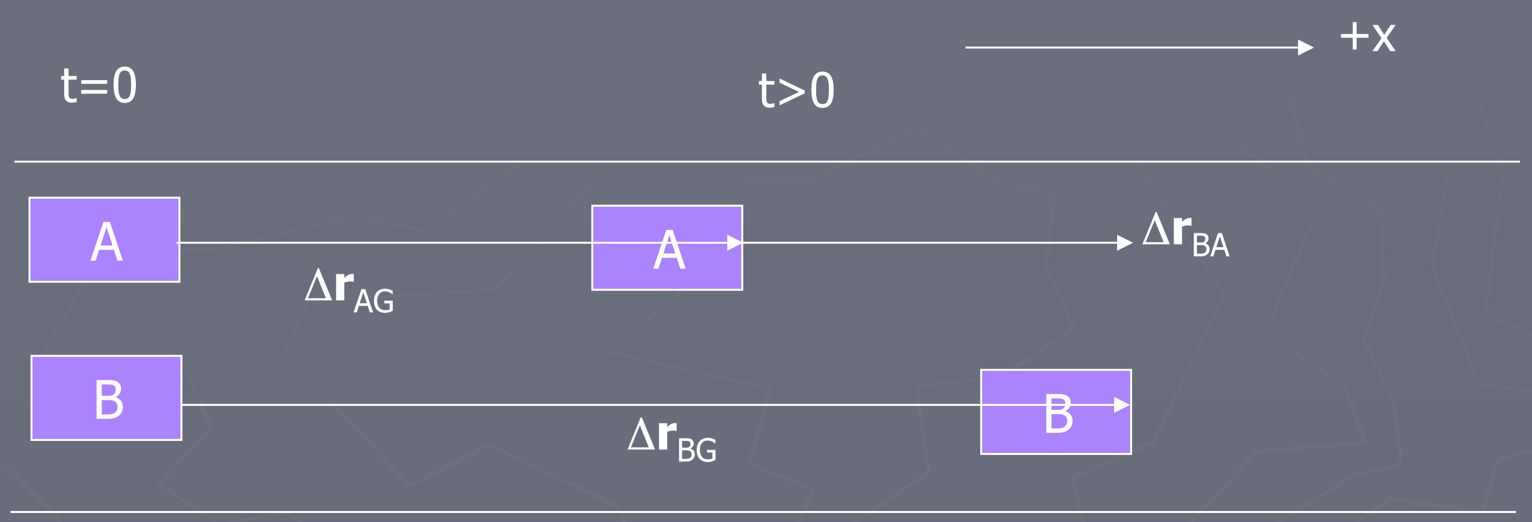
$$\Delta x = v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2$$

$$= v_{ix}\Delta t + 0 = 35.3\ \text{m}$$

Velocity is Relative!

Example: You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour east. What is the speed of car B relative to car A?

Example continued:



From the picture:

$$\Delta \mathbf{r}_{BG} = \Delta \mathbf{r}_{AG} + \Delta \mathbf{r}_{BA}$$

$$\Delta \mathbf{r}_{BA} = \Delta \mathbf{r}_{BG} - \Delta \mathbf{r}_{AG}$$

Divide by Δt:

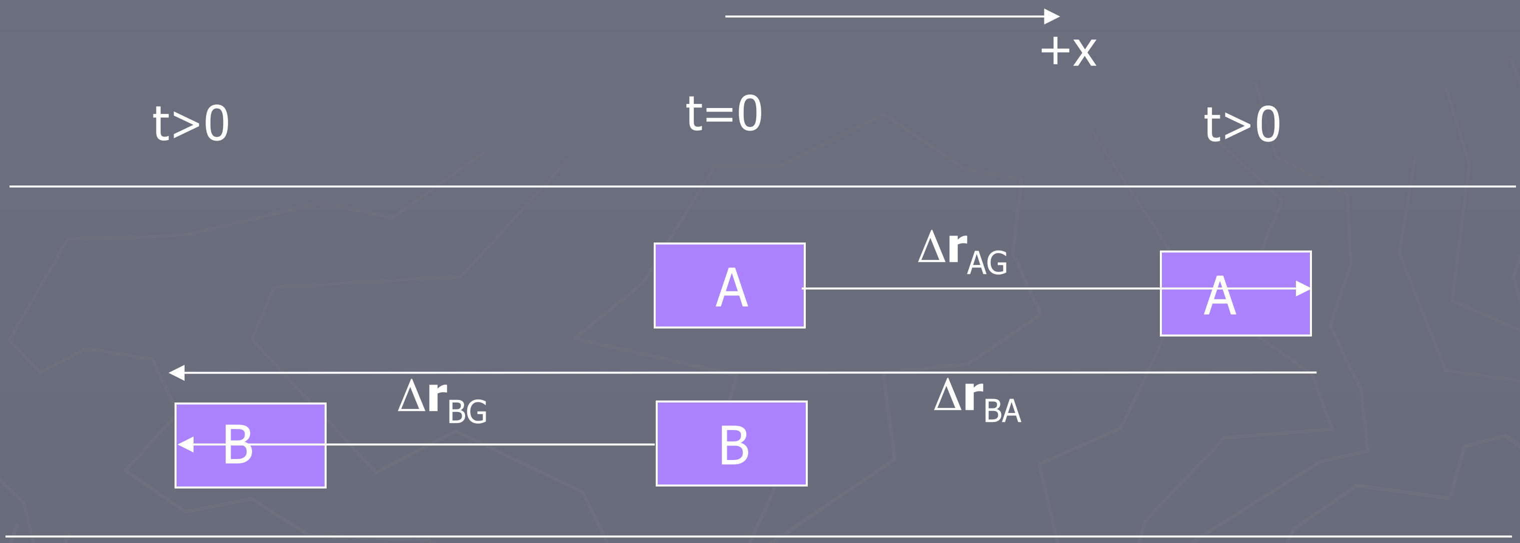
$$\mathbf{v}_{BA} = \mathbf{v}_{BG} - \mathbf{v}_{AG}$$

$$\mathbf{v}_{BA} = 65 \text{ miles/hr east} - 60 \text{ miles/hr east}$$

$$= 5 \text{ miles/hour east}$$

Example: You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour west. What is the speed of car B relative to car A?

Example continued:



From the picture: $$\Delta \mathbf{r}_{BA} = \Delta \mathbf{r}_{BG} - \Delta \mathbf{r}_{AG}$$

Divide by Δt: $$\mathbf{v}_{BA} = \mathbf{v}_{BG} - \mathbf{v}_{AG}$$

$$= 65 \text{ miles/hr west} - 60 \text{ miles/hr east}$$

$$= 125 \text{ miles/hr west}$$